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SOME ESTIMATION METHODS FOR FIXED AND RANDOM PANEL DATA MODELS WITH SERIALLY CORRELATED ERRORS WITH APPLICATION

Musaab Amer Rajab
Mosab.Aamer1201a@coadec.uobaghdad.edu.iq
Assistant Chief Statistician, Ministry of Electricity, Administrative office, Department of Informatics and Systems, Baghdad, Iraq
Supervisor : Prof. Dr. Mohammed Sadeq Abdul.Razaq
dr_aldouri@coadec.uobaghdad.edu.iq
Department of Statistics, Faculty of Administration and Economics, Baghdad University, Baghdad, Iraq
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Abstract:

This paper includes the study of panel data models with fixed and random parameters when the errors are serially correlated of the first order, the parameters of these models were estimated by two methods, the feasible lest square method (FGLS) and the mean group method (MG), efficiency comparison of these estimators were made when the regression parameters were fixed and random. Real data were applied to evaluate both the (FGLS) method performance and the (MG) method on the two models, and the (MAPE) scale was used to compare the efficiency of the estimators. The results showed that for the panel data model with fixed parameter, the (FGLS) method is better in in estimating the parameters then the (MG) method. As for the panel data model with random parameters, the results showed that the (FGLS) method in the case of the (Swamy) model is better in estimating than the (MG) method.

Keywords: FGLS estimation method, Fixed stochastic parameter regression model, Random stochastic parameter regression model, (MG) estimation method.

1. INTRODUCTIONS:

To study any of the economic, social, medical or other phenomena that the researcher chooses in his study, he must provide data for that phenomenon from solid and reliable sources, when studying a specific phenomenon during a specific time period, time serious data must be because collected, and this serious may include an autocorrelation problem because it is unstable, In this case, the general least square (GLS) method should be used to estimate the model parameters.

And when studying a certain phenomenon for several sectors of different groups, it is necessary to collect cross-sectional data, which in most cases is a problem of heterogeneity of error variance, so the weighted least square method (WLS) should be used to estimate the modeling parameters. And those random errors in both types of data above are considered the main reason for the occurrence of problems in the data. Instead of analyzing each type of data above separately, in which the researcher may obtain inefficient estimates, it required obtaining another type of data by merging the two types of data above and obtaining what is called Panel Data. Most of the research replied on estimating the parameters and testing them for the panel data on two methods: the generalized least square (GLS) when the variance-covariance matrix is known and the (FGLS) method when the variance-covariance matrix is unknown, and it is one of the methods adopted in this research to estimate the model parameters [5, 12].

For example, the phenomenon of the spread of a particular disease in a particular country classified according to the regions or cities in that country and measured for a specific period, accordingly, the observations of this phenomenon at the level of each city represent the cross-sectional data, while the observations during a period of time for each city and during a certain period of time represent the time series data For example, the phenomenon of the spread of a particular disease in a particular country classified according to the regions or cities in that country and measured for a specific period, accordingly, the observations of this phenomenon at the level of each city represent the cross-sectional data, while the observations during a period of time for each city and during a certain period of time represent the time series data[3].

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And the importance of statistical analysis of this type of data is to assess the effects of the explanatory variables on the dependent variable during the specified time period, and the efficient estimation of the model parameters is a major goal in the analysis of the dual data, and that the data collection process in this way leads to obtaining accurate parameters that represent the study population in a way Reliable and correct, due to taking into account the time factor and the existence of a correlation between the sample items[12].

In this research, the parameters of two models will be estimated, namely the panel data regression model with fixed parameters and the panel data regression model with random parameters, as follows:

First/ the panel data regression model with fixed parameters, which is related to the fixed limit in the model, through changes or differences that occur in the fixed limit through cross sections with constant change marginal tendencies in the model.

Secondly/ the panel data regression model with random parameters, which is related to the marginal tendencies in the model, through the changes or differences the occur in the marginal tendencies of the cross-sections with the constant limit in the models, and this type of models gives a more detailed analysis than the first type, and the researcher indicated that (Swamy) through a model named after him using the general least squares (GLS) method to estimate the parameters of his model due to the correlation between the cross-sections.

2. Fixed parameter regression of panel data model:

In the non-random (fixed) parameter regression model, differences in fixed terms across the segments are assumed through cross-sections and through time series and the errors are:

Cross sections with heterogeneity of variance in addition to being serially correlated of the first order. And with there for, individuals are selected cross sections from a population that has a vector of common regression parameters ($\bar{\beta}$), i.e. [11].

$\beta_1 = \beta_2 = \cdots = \beta_N = \bar{\beta}$

Suppose that the variable (Y) of the cross-sectional unit (i_{th}) in the time serious (t) is specified as a linear function (K) of the explanatory variables (X_{kit}) in the following form:

$y_{it} = x_{it}\beta_i + u_{it},\ i = 1,2,\ldots,N;\ t = 1,2,\ldots,T$ ... (1)

Whereas:

y_{it}: represent the observation (t) of the observation of the dependent variable of the cross section (i).

β_i: is an array vector of order (1*K) containing the regression parameters $(\beta_1, \ldots, \beta_K)$ of the cross section regression model (i).

x_{it}: Vector explanatory variables of the cross sectional regression model (i) including observations (t) it is ranked (k*1).

u_{it}: represents the random error (t) for the slop of the cross-section (i).

From equation (1), the following can be obtained:

$y_i = X_i\beta_i + u_i$ ... (2)

Whereas

y_i: Vertical vector of order (T*1) from the observation of the independent variable for the cross section (i).

X_i: Matrix of (T*K) order observations of explanatory variables for cross section (i).

β_i: Vertical vector of order (K*1) of regression parameters of cross section (i).

u_i: Vertical vector of order (T*1) of random errors of cross section (i).

$$y_i = (y_{i1}, \ldots, y_{iT})', X_i = (x'_{i1}, \ldots, x'_{iT})', \beta_i = (\beta_{i1}, \ldots, \beta_{iK})', u_i = (u_{i1}, \ldots, u_{iT})'$$

The model can be adopted for (N) cross-section, where the number of observations becomes (n=NT). When the performance of a single individual (across-section of the data base) is significant, the equation of separate regression for each cross section can be estimated using the ordinary least squares (OLS) method. And the estimator (OLS) for (β_i) is obtained according to the formula:

$\hat{\beta}_i = (X'_iX_i)^{-1}X'_iy_i$ ... (3)

According to the following assumption, (β_i) is the best unbiased linear estimator (BLUE) for (β_i):

- The first assumption: the expectation of errors is zero
$$E(u_i) = 0; \forall i = 1,2,\ldots,N$$
- The second assumption: the faults have the same variance for each cross-section
$$E(u_iu'_j) = \begin{cases} \sigma_u^2 I_T & if\, i = j \\ 0 & if\, i \neq j \end{cases} \quad i,j = 1,2,\ldots,N$$
- The third assumption: is that the explanatory variables are not random, that is, fixed in repeated samples, and therefore not associated with errors, and also that:

Rank (Xi) = K < T ; $\forall\ i = 1,2,\ldots,N$

And the best unbiased linear estimator (BLUE) for $(\bar{\beta})$ under assumption from (1) to (3) is:

$\hat{\bar{\beta}}_{PLS-SC} =$
$(X'V^{-1}X)^{-1}(X'V^{-1}Y)$ ... (4)

$whereas$

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$$V=\begin{pmatrix}\sigma_{\varepsilon_1}^2\Omega_{11} & 0 & \cdots & 0\\ 0 & \sigma_{\varepsilon_2}^2\Omega_{22} & \ddots & \vdots\\ \vdots & \ddots & \ddots & 0\\ 0 & \cdots & 0 & \sigma_{\varepsilon_N}^2\Omega_{NN}\end{pmatrix}$$
...(5)

And that

$$\Omega_{ii}=\frac{1}{1-\phi_i^2}\begin{pmatrix}1 & \phi_i & \phi_i^2 & \cdots & \phi_i^{T-1}\\ \phi_i & 1 & \phi_i & \cdots & \phi_i^{T-2}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \phi_i^{T-1} & \phi_i^{T-2} & \phi_i^{T-3} & \cdots & 1\end{pmatrix} \quad \text{... (6)}$$

To make the ($\hat{\bar{\beta}}_{PLS-SC}$) estimator feasible, the following consistent estimators of (ϕ_i) and ($\sigma_{\varepsilon_i}^2$) are used:

$$\hat{\phi}_i=\frac{\sum_{t=2}^{T}\hat{u}_{it}\hat{u}_{i,t-1}}{\sum_{t=2}^{T}\hat{u}_{i,t-1}^2} \quad \text{... (7)}$$

$$\hat{\sigma}_{\varepsilon_i}^2=\frac{\hat{\varepsilon}_i'\hat{\varepsilon}_i}{T-K'} \quad \text{... (8)}$$

$$\hat{u}_i=(\hat{u}_{i1},\dots,\hat{u}_{iT})'$$

$$\hat{u}_i=y_i-X_i\hat{\beta}_i\ ,$$

$$\hat{\beta}_i=(X_i'X_i)^{-1}X_i'y_i\ ,$$

$$\hat{\varepsilon}_i=(\hat{\varepsilon}_{i1},\hat{\varepsilon}_{i2},\dots,\hat{\varepsilon}_{iT})'$$

$$\hat{\varepsilon}_{i1}=\hat{u}_{i1}\sqrt{1-\hat{\phi}_i^2}$$
... (9)

$$\hat{\varepsilon}_{it}=\hat{u}_{it}-\hat{\phi}_i\hat{u}_{i,t-1} \qquad \textit{for}\ t=2,\dots,T$$
... (10)

This estimator is called pooled least square with serial correlation (PLS_SC).

Process can be viewed as building a single model to describe the entire group segmented individuals (cross-sections) instead of building a separate model for each of them. Again, we assumptions (1) to (3) are satisfied and odd the following assumption:

- The fourth assumption: individuals (cross-sections) in the data base are selected from a population with a common regression vector ($\bar{\beta}$), i.e.
  $\beta_1=\beta_2=\cdots=\beta_N=\bar{\beta}$

Under this assumption, observations can be aggregated for each individual (cross-section), and a single regression can be performed to obtain the effective estimator for ($\bar{\beta}$), now the system of the equation writes as follows:

$$Y=X\bar{\beta}+u \quad \text{... (11)}$$

Whereas

$$,\ X=(X_1',\dots,X_N')',u=\left(u_1',\dots,u_N'\right)'Y=(y_1',\dots,y_N')'$$

$\bar{\beta}=\left(\bar{\beta}_1,\dots,\bar{\beta}_K\right)'$ is the vector of the fixed features to be estimated.

We will differentiate between two cases to estimate ($\bar{\beta}$) in hypothesis (4) based on variance-covariance of (u):

In the first case, the faults have the same variance for each individual cross-section as given by assumption (2). In this case, the effective and unbiased estimator for ($\bar{\beta}$) under assumptions from (1) to (4) is:

$$\hat{\bar{\beta}}_{CP-OLS}=(X'X)^{-1}X'Y \quad \text{... (12)}$$

This estimator is called the classical pooling with ordinary least square (CP_OLS).

In the second case, errors have different variances at the individual level and are synchronously correlated as in a frame work (SUR):

- Fifth Assumption:
  $$E\left(u_iu_j'\right)=\begin{cases}\sigma_{ii}I_T & \text{if } i=j\\ \sigma_{ij}I_T & \text{if } i\neq j\end{cases} \qquad i,j=1,2,\dots,N$$

Under assumption (1), (3), (4), (5), the effective and unbiased CP estimator for ($\bar{\beta}$) is:

$$\hat{\bar{\beta}}_{CP-SUR}=[X'(\Sigma_{sur}\otimes I_T)^{-1}X]^{-1}[X'(\Sigma_{sur}\otimes I_T)^{-1}Y]$$
... (13)

Classical Pooling with Seemingly unrelated regression (CP_SUR)

Whereas

$$\text{... (14)}\ \Sigma_{sur}=\begin{pmatrix}\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N}\\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N}\\ \vdots & \vdots & \ddots & \vdots\\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN}\end{pmatrix}$$

To make this this ($\hat{\bar{\beta}}_{CP-SUR}$) estimator feasible ($\sigma_{ij}$) can be replaced by the following unbiased and consistent estimator:

$$\hat{\sigma}_{ij}=\frac{\hat{u}_i'\hat{u}_j}{T-K};\quad \forall i,j=1,2,\dots,N \quad \text{... (15)}$$

Whereas

$\hat{u}_i=y_i-X_i\hat{\beta}_i$ Residuals vector obtained from the application of (OLS) [2].

3. Stochastic parameter regression of panel data models (SPR)

Suppose there are observations to (N) cross-sections for (T) time serious, and we assume that the variable (Y) for unit (i) at time (t) is determined as a linear function for (K) of completely independent variables (X_{kit}) in the following form [11]:

$$y_{it}=x_{it}\beta_i+u_{it},\qquad i=1,\dots,N;\quad t=1,\dots,T$$
... (16)

u_{it}: represents the random error

x_{it}: represent the vector of independent variables (1*K)

β_i: represent the parameters vector of the (K*1) regression model

if the performance of a single individual (one cross-section) from the database is important, a separate regression equation can be estimated for each unit

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separately and then rewrite the model in (16) as follows:

$y_i = X_i\beta_i + u_i; \quad i = 1, \dots, N$ **...** (17)

y_i: A vector of (T*1) order from the observations of the dependent variable for section (i).

X_i: a matrix of degree (T*K) from the observations of explanatory variables for section (i).

β_i: A vector of (K*1) order for the unknown parameters (liminal slopes) for section (i).

u_i : A vector of order (T*1) for random errors for section (i).

, $X_i = (x'_{i1}, \dots, x'_{iT})', \beta_i = (\beta_{i1}, \dots, \beta_{iK})' \ \ y_i = (y_{i1}, \dots, y_{iT})'$ $, u_i = (u_{i1}, \dots, u_{iT})'$

We assume the model in (16) & (17) under the following assumption:

- **The first assumption**: the expectation of error is zero
  $E(u_i) = 0; \quad \forall i = 1, \dots, N$
- **the second assumption**: the explanatory variables are not random (in repeated samples), then we assume independence with other variables in the model and the value of the rank:
  $(X_i'X_i) = K; \quad \forall i = 1, \dots, N, \quad \textit{where } K < T, N$
- **the third assumption**: the errors have a fixed variation for each individual (cross-section), but there is a problem of heterogeneity of the variation in the cross section, in addition to being serially interconnected of the first degree, meaning that the randomness error for each period is linearly dependent on the random error of the preceding periods [3,p319].
  $u_{it} = \phi_i u_{i,t-1} + \varepsilon_{it}; \quad |\phi_i| < 1,$ where ϕ_i *for* $i = 1, \dots, N$

The coefficients are serially correlated of the first order and are stable.

Whereas

$$E(\varepsilon_{it}) = 0,$$

$E(u_{i,t-1}\varepsilon_{jt}) = 0; \quad \forall i, j, \textit{ and } t. \textit{ And}$

$E(\varepsilon_{it}\varepsilon_{js}) = \begin{cases} \sigma^2_{\varepsilon_i} & \textit{if } t = s; i = j \\ 0 & \textit{otherwise} \end{cases} \quad i, j = 1, \dots, N; \quad t, s = 1, \dots, T$

Assumed that errors in the initial or initial time serious have the same characteristics as in the subsequent periods, so we assume that [3, p320]:

$E(u^2_{i0}) = \sigma^2_{\varepsilon_i}/1 - \phi^2_i; \quad \forall i$

- **the fourth assumption:** the regression model feature vector is determined as follows:
  $\beta_i = \bar{\beta} + \pi_i$ … (18)
  Whereas
  $\bar{\beta} = (\bar{\beta}_1, \dots, \bar{\beta}_K)'$
  $\bar{\beta}$: It is a vector of non-random parameters (fixed) of order (K*1) that is determined by the method of least square (OLS).
  And that
  $\pi_{\mathrm{i}} = (\pi_{\mathrm{i}1}, \dots, \pi_{\mathrm{iK}})'$
  π_{i}: Vector of random errors for the parameters of order (K*1)
  β_i: Vector of order (K*1)
  And that

$E(\pi_i\pi_j') = \begin{cases} \gamma^* & \textit{if } i = j \\ 0 & \textit{if } i \neq j \end{cases} \quad i, j = 1, \dots, N; \quad k = 1, \dots, K$

Where (γ^*) is the diagonal matrix and is equal to

$\gamma^* =$ *For* k*=1,…, K* $diag\{\gamma^*_k\}$

We also assume that

$\mathrm{E}(\pi_i u_{jt}) = 0 \quad \forall i \textit{ and } j$

Using the fourth assumption, the model can be rewritten in (17) and given that the parameters are random, the model becomes the following form, which is called the (Swamy) model.

$y_i = X_i\bar{\beta} + e_i$ … (19)

$e_i = Z_i\pi_i + u_i$

And by writing the (Swamy) model in (19) for (n) cross-sections using matrices as follows:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \bar{\beta} + \begin{bmatrix} x_1 & 0 & 0 & 0 \\ 0 & x_2 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & x_n \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \vdots \\ \pi_n \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

$Y = X\bar{\beta} + Z\pi + u$ … (20)

e= $Z\pi + u$

Whereas

Y: A vector of the order (N*1) from observation of the dependent variables for all cross-sections.

X: matrix of degree (nT*k) from observations of explanatory variables for all cross-sections.

$\bar{\beta}$: A vector of the order (K*1) of the unknown parameters of the random parameter panel data regression model.

Z: A matrix of degree (nT*nK) is a diagonal matrix whose elements are Xi (i=1,2,…,n) defined in model (17).

π: A vector of order (nK*1) from the random errors of the parameters defined in (18).

u: A vector of order (nT*1) from the random errors of model (20).

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$X = (X_1', \dots, X_N')', u = (u_1', \dots, u_N')', \pi = Y = (y_1', \dots, y_N')', (\pi_1', \dots, \pi_N')'$

$Z = diag\{X_i\}$; *for* $i = 1, \dots, N$

Under assumptions from (1) to (4), the best unbiased linear estimator (BLUE) for ($\bar{\beta}$) is:

$\hat{\hat{\beta}}_{SPR-SC} = (X'\Lambda^{*-1}X)^{-1}X'\Lambda^{*-1}Y;$ … (21)

And its variance-covariance matrix is

$var\left(\hat{\hat{\beta}}_{SPR-SC}\right) = (X'\Lambda^{*-1}X)^{-1}$ … (22)

Whereas

$\Lambda^* = V + Z(I_N \otimes \gamma^*)Z'$ … (23)

And that

(V) Defined in (5)

(Ω_{ii}) Defined in (6)

And that

$\gamma^* = \left[\frac{1}{N-1}\left(\sum_{i=1}^N \beta_i^*\beta_i^{*'} - \frac{1}{N}\sum_{i=1}^N \beta_i^* \sum_{i=1}^N \beta_i^{*'}\right)\right] - \frac{1}{N}\sum_{i=1}^N \sigma_{\varepsilon_i}^2 (X_i'\Omega_{ii}^{-1}X_i)^{-1}$ … (24)

Whereas

$\beta_i^* = (X_i'\Omega_{ii}^{-1}X_i)^{-1}X_i'\Omega_{ii}^{-1}y_i$ … (25)

To make the ($\hat{\hat{\beta}}_{SPR-SC}$) estimator feasible, the formatted estimates are used for (ϕ_i , $\sigma_{\varepsilon_i}^2$) as in formula (7) and (8).

And that

$\hat{u}_i = (\hat{u}_{i1}, \dots, \hat{u}_{iT})'$

$\hat{u}_i = y_i - X_i\hat{\beta}_i$,

$\hat{\beta}_i = (X_i'X_i)^{-1}X_i'y_i$

While

$\hat{\varepsilon}_i = (\hat{\varepsilon}_{i1}, \hat{\varepsilon}_{i2}, \dots, \hat{\varepsilon}_{iT})'$

And that ($\hat{\varepsilon}_{it}$) and ($\hat{\varepsilon}_{i1}$) are shown in formula (9) and (10).

- **Standard Stochastic parameter mode (Swamy's Model)** [11]

  In the standard stochastic parameter model presented by (Swamy)[14], I assume that the errors are cross-sections in which there is a problem of heterogeneity of variance and that they are sequentially independent with respect to the parameter. I assume the same conditions in assumption (4). Therefore, the best linear unbiased estimator (BLUE) for ($\bar{\beta}$), according to (Swamy) [14] is:

  $\hat{\beta}_{SPR} = (X'\Lambda^{-1}X)^{-1}X'\Lambda^{-1}Y$ … (26)

  Whereas

  $\Lambda = (\Sigma_H \otimes I_T) + Z(I_N \otimes \gamma)Z'$ … (27)

  $\Sigma_H = diag\{\sigma_i^2\}$; *for* $i = 1, \dots, N,$

  $\sigma_i^2 = var\ (u_i)$

  And that (γ) in this estimator is equal to (γ^*) in given ($\Omega_{ii} = I_T$) and that (i=1,…,N):

$\gamma = \left[\frac{1}{N-1}\left(\sum_{i=1}^N \beta_i\beta_i' - \frac{1}{N}\sum_{i=1}^N \beta_i \sum_{i=1}^N \beta_i'\right)\right] - \left[\frac{1}{N}\sum_{i=1}^N \sigma_i^2 (X_i'X_i)^{-1}\right]$ … (28)

To make the ($\hat{\beta}_{SPR}$) estimator feasible use (Swamy) [27] the unbiased and consistent estimator for (σ_i^2) as in formula (8).

For the purpose of calculating the sub-parameters for each cross-section, i.e. for each country:

The best unbiased linear BLUE estimator in stochastic parameter Regression with serial correlation ($\hat{\hat{\beta}}_{SPR-SC}$) is [1]:

$$\hat{\hat{\beta}}_{SPR-SC} = (X'\Lambda^{*-1}X)^{-1}X'\Lambda^{*-1}Y \quad \dots (29)$$

And the variance - covariance matrix is:

$$var\left(\hat{\hat{\beta}}_{SPR-SC}\right) = (X'\Lambda^{*-1}X)^{-1} \quad \dots (30)$$

Whereas:

$$\Lambda^* = V + Z(I_N \otimes \gamma^*)Z' \quad \dots (31)$$

$$V = \begin{pmatrix} \sigma_{\varepsilon_1}^2\Omega_{11} & 0 & \cdots & 0 \\ 0 & \sigma_{\varepsilon_2}^2\Omega_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{\varepsilon_N}^2\Omega_{NN} \end{pmatrix} \quad \dots (32)$$

$$\Omega_{ii} = \frac{1}{1-\phi_i^2}\begin{pmatrix} 1 & \phi_i & \phi_i^2 & \cdots & \phi_i^{T-1} \\ \phi_i & 1 & \phi_i & \cdots & \phi_i^{T-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_i^{T-1} & \phi_i^{T-2} & \phi_i^{T-3} & \cdots & 1 \end{pmatrix} \quad \dots (33)$$

And that

$$\gamma^* = \left[\frac{1}{N-1}\left(\sum_{i=1}^N \beta_i^*\beta_i^{*'} - \frac{1}{N}\sum_{i=1}^N \beta_i^* \sum_{i=1}^N \beta_i^{*'}\right)\right] - \frac{1}{N}\sum_{i=1}^N \sigma_{\varepsilon_i}^2 (X_i'\Omega_{ii}^{-1}X_i)^{-1} \quad \dots (34)$$

$$\beta_i^* = (X_i'\Omega_{ii}^{-1}X_i)^{-1}X_i'\Omega_{ii}^{-1}y_i \quad \dots (35)$$

To make the ($\hat{\hat{\beta}}_{SPR-SC}$) estimator feasible, the following consistent estimates of (ϕ_i) and ($\hat{\sigma}_{\varepsilon_i}^2$) are used:

$$\hat{\phi}_i = \frac{\sum_{t=2}^T \hat{u}_{it}\hat{u}_{i,t-1}}{\sum_{t=2}^T \hat{u}_{i,t-1}^2} \quad \dots (36)$$

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$\widehat{\sigma}^2_{\varepsilon_i} = \frac{\hat{\varepsilon}_i'\hat{\varepsilon}_i}{T-K'}$ … (37)

And that

$\hat{u}_i = (\hat{u}_{i1}, \dots, \hat{u}_{iT})'$, $\hat{u}_i = y_i - X_i\hat{\beta}_i$, $\hat{\beta}_i = (X_i'X_i)^{-1}X_i'y_i$,

While

$\hat{\varepsilon}_i = (\hat{\varepsilon}_{i1}, \hat{\varepsilon}_{i2}, \dots, \hat{\varepsilon}_{iT})'$, $\hat{\varepsilon}_{i1} = \hat{u}_{i1}\sqrt{1-\hat{\phi}_i^2}$, $\hat{\varepsilon}_{it} = \hat{u}_{it} - \hat{\phi}_i\hat{u}_{i,t-1}$ for $t = 2, \dots, T$

It should be noted that ($\hat{\hat{\beta}}_{SPR-SC}$) can be rewritten as a weighted average estimator (GLS) for each cross section [1].

$\hat{\hat{\beta}}_{SPR-SC} = \sum_{i=1}^{N} W_i^*\beta_i^*$ ...(38)

And that

$W_i^* = \{\sum_{i=1}^{N} [\gamma^* + \sigma_{zi}^2(X_i'\Omega_{ii}^{-1}X_i)^{-1}]^{-1}\}^{-1}\left\{\sum_{i=1}^{N} \left[\gamma^* + \sigma_{\varepsilon_i}^2(X_i'\Omega_{ii}^{-1}X_i)^{-1}\right]^{-1}\right\}$ … (39)

It turns out that ($\hat{\hat{\beta}}_{SPR-SC}$) in formula (24) is a weighted average of the (OLS) estimates for a given cross-section. Finally, the formula (24) benefits from the fact that [9].

$(A + BDB')^{-1} = A^{-1} - A^{-1}BEB'^{A^{-1}} + A^{-1}BE(E + D)^{-1}EBA'^{-1}$ … (40)

And that

(A) And (D) are non-singular matrices of (m*n) degree, and (B) a matrix of (m*n) degree [62:P33]

E = $(B'A^{-1}B)^{-1}$[10:P33] … (41)

In addition to the estimation of ($\hat{\hat{\beta}}_{SPR-SC}$), the researcher often wishes to obtain estimations of the (β_i) vectors of cross-sections as well, if the interest is limited to the class of estimators (β_i^*) for which it is [8:p541]

E ($\hat{\beta}_i*/\beta_i$) = βi

And an estimator (OLS) for a single cross section (b_i) is appropriate. However, if there is no condition on (β_i), the best unbiased linear estimator is:

$\hat{\beta}_i = \hat{\beta} + \gamma^* x'_i\ (x_i\gamma^*x'_i + \sigma_{ii}I)^{-1}(y_i - X_i\hat{\beta})$

$\hat{\beta}_i = (\gamma^{*-1} + \sigma_{ii}{}^{-1}X_i'X_i)^{-1}(\sigma_{ii}{}^{-1}X_i'X_ib_i + \gamma^{*-1}\hat{\beta})$ … (42)

To obtain the variance ($\hat{\beta}_i$) , Green (1997,672) suggested the formula (30):

$\hat{\beta}_i = [A_i \quad (I - Ai)]\begin{bmatrix}\hat{\beta}\\ b_i\end{bmatrix}$ … (43)

Whereas:

$A_i = (\gamma^{*-1} + \sigma_{ii}{}^{-1}X_i'X_i)^{-1}\ \gamma^{*-1}$ … (44)

Var ($\hat{\beta}_i$) = $[A_i \quad (I - A_i)]$ Var$\begin{pmatrix}\hat{\beta}\\ b_i\end{pmatrix}\begin{bmatrix}A_i'\\ (I - A_i)'\end{bmatrix}$ … (45)

Whereas:

Var$\begin{pmatrix}\hat{\beta}\\ b_i\end{pmatrix}$ = $\begin{bmatrix}Var(\hat{\beta}) & Cov(\hat{\beta}, b_i)\\ Cov(\hat{\beta}, b_i) & Var(b_i)\end{bmatrix}$ … (46)

Estimator ($\hat{\beta}$) using the (GLS) method is consistent and effective, and according to (Lemma 2.1) in the source (Hausman) [7]

Asy. $Cov(\hat{\beta}, b_i)$ = Asy. $Var(\hat{\beta})$ – Asy. $Cov(\hat{\beta}, \hat{\beta} - b_i)$ = $Var(\hat{\beta})$

After doing some mathematical operations, we get:

Asy. $Var(\hat{\beta}_i) = Var(\hat{\beta}) + (I - A_i)\{Var(b_i) - Var(\hat{\beta})\}(I - A_i)'$

And to obtain the feasible estimations of the above formulas, each (σ_{ii}) It can be offset by an OLS estimate [9]:

$\widehat{\sigma_{ii}} = \frac{(y_i - X_ib_i)'(y_i - X_ib_i)}{T_i - K}$ … (47)

4. Feasible Generalized Least Square (FGLS)

The estimators of (MSPR-SC) need to estimate the elements of matrices (variance-covariance) because they are unknown and to make these estimators

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feasible, it is suggested to use the following consistent estimators: (ϕ_i) and $(\sigma^2_{\varepsilon_i})$ [11]:

$$\widehat{\phi}_i = \frac{\sum_{t=2}^{T} \hat{u}_{it}\hat{u}_{i,t-1}}{\sum_{t=2}^{T} \hat{u}^2_{i,t-1}} \quad \ldots (48)$$

$$\hat{\sigma}^2_{\varepsilon_i} = \frac{\hat{\varepsilon}_i'\hat{\varepsilon}_i}{T-K'} \quad \ldots (49)$$

Where

$\hat{u}_i = (\hat{u}_{i1}, \ldots, \hat{u}_{iT})' = y_i - X_i \hat{\beta}_i$, $\hat{\beta}_i = (X_i'X_i)^{-1}X_i'y_i$,

While $\hat{\varepsilon}_i = (\hat{\varepsilon}_{i1}, \hat{\varepsilon}_{i2}, \ldots, \hat{\varepsilon}_{iT})'$, $\hat{\varepsilon}_{i1} = \hat{u}_{i1}\sqrt{1-\hat{\phi}_i^2}$, $\hat{\varepsilon}_{it} = \hat{u}_{it} - \hat{\phi}_i \hat{u}_{i,t-1}$ *for* $t = 2, \ldots, T$

Replacing (ϕ_i) with $(\widehat{\phi}_i)$ in the matrix (Ω_{ii}) gives consistent estimates of (Ω_{ii}), we get $(\hat{\Omega}_{ii})$, and using $(\hat{\sigma}^2_{\varepsilon_i})$ and $(\hat{\Omega}_{ii})$ gives consistent estimates for (V) And (γ^*) which is $(\hat{V})$ and $(\hat{\gamma}^*)$ and using the coherent estimators $(\hat{\gamma}^*, \hat{\Omega}_{ii}, \hat{\sigma}^2_{\varepsilon_i})$ we get a coherent estimator for (Λ^*) which is $(\hat{\Lambda}^*)$ and using $(\hat{\Lambda}^*)$ we obtain a feasible estimator for $(\hat{\hat{\beta}}_{SPR-SC})$.

In short, using the $(\hat{V})$ defined above results in a possible (PLS-SC) estimator, For the (SPR) estimator, Swamy [15] used the following unbiased and consistent estimator for σ_i^2: $\hat{\sigma}_i^2 = \frac{\hat{\varepsilon}_i'\hat{\varepsilon}_i}{T-K'}$ where $(\hat{\varepsilon}_i)$ is specified in (7).

5. Mean Group Estimator (MG)

Suggest (Abo Nazel)[2, 1], use an estimator (MG) as an alternative estimator for the general random regression model is defined as follows:

$$\bar{\beta}_{SMG} = \frac{1}{N}\sum_{i=1}^{N} \hat{\beta}_i \quad \ldots (50)$$

You notice that this estimate is average of the ordinary least square (OLS) estimates, which is $(\hat{\beta}_i)$.

And for easy verification the (MG) estimator is constant with $(\bar{\beta})$ when both are $(N, T \rightarrow \infty)$, (Abonazel) [1] showed the statistical properties of the (MG) estimator.

6. DESCRIPTION OF DATA

The data studied in this paper represent the per capita share of electric energy consumption as a dependent variable and the mean explanatory variables affecting it, which are the per capita share of the gross domestic product and the consumer price index, it is panel data that includes five countries, namely Iraq, and its comparison with neighboring countries (N=5), which represent cross-sections measured over a period of nine years (T=9) which in turn represent the time serious.

Global ESCWA committee of the United Nations and ESCWA is among the committees that work under the supervision of the economic and social council, and the ESCWA committee was established by the economic commission for western Asia in order to stimulate the economic activity of the number countries.

7. ANALYSIS OF DATA

In this section, the results of real data analysis will be presented, which were represented by the per capita share of electric energy consumption (Y), the consumer price index (X1), and the per capita gross domestic product (X2) for five countries, the estimations of the parameters of the model were extracted and compared between them through the measure the mean absolute percentage error (MAPE).

- **First Stage:**

  At this stage, the MAPE scale and parameter estimators were extracted for the two fixed and random models, as follows:

  - **First/ fixed parameter regression of panel data models:**

    As for the best method in the fixed parameter estimation model and through the (MAPE) scale, we note that the feasible general least squares method in the case of the pooled estimator with serial correlation (PLS_SC) is the best method for estimating the fixed parameter, followed by the (CP_SUR) method when the contrasts are different between the cross-sections (countries) and interconnected simultaneously and finally the method (MG) as shown in table (1).

Table (1)
Shows the preference of the estimation methods in the case Of the fixed parameters model using the (MAPE) scale

Methods Scale	FGLS			MG
	PLS_SC	CP_OLS	CP_SUR	
MAPE	0.308274	0.326428	0.334715	0.360112

As for the values of the parameters estimates according to the fixed parameters estimation model, they are shown as in table (2), and the estimators were calculated for the (FGLS) method for the estimator (PLS_SC), it was calculated

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according to formula No. (4), and the estimator (CP_OLS) were calculated according to formula No. (12), and the estimator (CP_SUR) were calculated according to formula No. (13), and for the estimator according to the (MG) method, it was calculated according to formula No.(50).

Table (2)

It shows the parameter values in the case of fixed parameters model according to the estimation methods

Methods parameters	FGLS			MG
	PLS_SC	CP_OLS	CP_SUR	
β_0	620.1319	39.78848	-288.627	-294.797
β_1	3.233839	3.218214	1.369212	9.433515
β_2	0.185533	0.344356	0.480047	0.280955

- **Second/ Stochastic parameter regression of panel data models:**

With regard to the best method in the model of estimating stochastic parameters and through the (MAPE) scale, we note that (Swamy's) estimation method is the best, followed by serial correlation stochastic parameter estimation method (SPR_SC) and the (MG) method came last, as shown in table No. (3).

Table (3)

Shows the preference of the estimation methods in the case Of the stochastic parameter model using the (MAPE) scale

methods scale	FGLS		MG
	SPR_SC	SPR (Swamy)	
MAPE	0.350622	0.34655	0.360112

As for the values of the feature estimates according to the stochastic parameters estimation model, the estimators were calculated according to the estimation methods. For the (FGLS) method, the estimator (SPR_SC) was calculated according to formula (21), and the estimator (SPR) or (Swamy) according to formula (26), and for the estimation method mean group (MG), the estimator was calculated according to formula (50), and it is shown as in table (4).

Table (4)

It shows the parameter values in the case of stochastic parameters model according to the estimation methods

Methods parameter	FGLS		MG
	SPR_SC	SPR (Swamy)	
β_0	-273.544	-240.839	-294.797
β_1	6.876592	6.039722	9.433515
β_2	0.338812	0.350719	0.280955

- **Second Stage:**

At this stage, the parameters of the fixed and stochastic parameters panel data model were estimated for each cross section (country) separately in terms of studying the per capita consumption of electric energy for Iraq and some of its neighboring countries. The results for each country will be mentioned below.

For **all countries** and through the (MAPE) scale, and in the case of the panel data model for **fixed parameters**, we note that the ordinary least squares (OLS) estimation method is the best compared to the mean group estimation method (MG) as shown in Table (5).

Table (5)

The preference for estimation methods is shown for the fixed parameter panel data model for all countries using the (MAPE) scale

Method Measure	Iraq		Egypt		Jordon		Morocco		Tunisia	
	(OLS)	(MG)	(OLS)	(MG)	(OLS)	(MG)	(OLS)	(MG)	(OLS)	(MG)
MAPE	0.0658	0.0835	0.0124	0.0764	0.0106	0.0192	0.0378	0.064	0.0081	0.0615

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As for the values of the parameters estimates for all countries according to the panel data model for fixed parameters, the estimator for the usual least squares method was calculated according to formula No. (3), while the estimator for the group mean (MG) method was calculated according to the formula (50), and as shown in Table No. (6).

Table (6)
It shows the parameter values in the case of fixed parameters model according to the estimation methods

methods / parameters	Iraq		Egypt		Jordon		Morocco		Tunisia	
	OLS	MG	OLS	MG	OLS	MG	OLS	MG	OLS	MG
β_0	-1504.64	-344.15	-1193.25	396.32	3214.47	102.74	-272.60	-1075.51	-1222.18	-553.39
β_1	50.18578	9.4335	-2.02461	9.4336	0.280797	9.4335	-1.5347	9.4335	0.26027	9.4335
β_2	-0.3567	0.28096	1.11818	0.28096	-0.38581	0.2810	0.3717	0.2810	0.657419	0.2810

For **all countries** and through the (MAPE) scale, and in the case of the panel data model for **stochastic parameters**, we note that the mean group (MG) estimation method is the best way to estimate model parameters compared to (SPR_SC) and (SPR (Swamy)) estimates as shown in table (7)

Table (7)
The preference for estimation methods is shown for the fixed parameter panel data model for all countries using the (MAPE) scale

Methods / scale	Iraq				Egypt				Jordon			
	FGLS SPR_SC	MG	FGLS SPR (Swamy)	MG	FGLS SPR_SC	MG	FGLS SPR (Swamy)	MG	FGLS SPR_SC	MG	FGLS SPR (Swamy)	MG
MAPE	1.859	0.086	1.714	0.086	0.486	0.051	0.4499	0.043	2.552	0.019	2.6101	0.019

Methods / scale	Morocco				Tunisia			
	FGLS SPR_SC	MG	FGLS SPR (Swamy)	MG	FGLS SPR_SC	MG	FGLS SPR (Swamy)	MG
MAPE	1.69253	0.062	1.5709	0.061	0.447	0.042	0.4835	0.035

As for the values of the estimates of the parameters for all countries according to the panel data model of **random parameters**, the estimators of this model were calculated according to the formula (43), while the estimator of the mean group (MG) method was calculated according to the formula (50), and as shown in Table (8)

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Table (8)
It shows the parameter values in the case of stochastic parameters model according to the estimation methods

Methods / parameter	Iraq				Egypt				Jordon			
	FGLS	MG	FGLS	MG	FGLS	MG	FGLS	MG	FGLS	MG	FGLS	MG
	SPR_SC		SPR (Swamy)		SPR_SC		SPR (Swamy)		SPR_SC		SPR (Swamy)	
β_0	-2018.86	-347.98	-1701.48	-322.76	-552.33	398.17	-820.58	414.49	2740.07	145.97	2946.72	175.93
β_1	34.113	6.689	31.382	6.0397	-0.5474	6.6894	-1.1447	6.0397	1.37947	6.6894	1.1122	6.0397
β_2	0.10998	0.342	0.0997	0.3507	0.851	0.3420	0.9626	0.3507	-0.2762	0.3420	-0.3296	0.3507

Methods / scale	Morocco				Tunisia			
	FGLS	MG	FGLS	MG	FGLS	MG	FGLS	MG
	SPR_SC		SPR (Swamy)		SPR_SC		SPR (Swamy)	
β_0	-327.52	-1004.24	-274.74	-967.83	-1103.71	-533.45	-1199.66	-504.02
β_1	-1.881	6.6894	-1.44897	6.0397	0.38241	6.6894	0.29862	6.0397
β_2	0.398	0.3420	0.36975	0.3507	0.62674	0.3420	0.65122	0.3507

8. CONCLUSION

In this research, after examining the estimations of the panel data model for fixed parameters using two estimation methods (MG) and (FGLS) when the errors are serially correlated of the first order, and after applying the real data of the per capita consumption of electric energy for Iraq and some Arab countries, the results indicated that The (PLS_SC) estimator using the Feasible Generalized Least Squares (FGLS) method has the lowest (MAPE) values than the Mean Group Estimation (MG) method, we conclude that the (FGLS) method is the best method for estimating the parameters of the fixed parameter model.

As for the estimators of the panel data model for random parameters, the results showed that the (Swamy) estimator, which is (SPR) using the (FGLS) method, has the lowest values for measuring efficiency (MAPE) than the mean group estimation method (MG), and therefore the (FGLS) method is the best way to estimate Random parameter model parameters.

The results of applying the real data for each cross-section (country) to the fixed parameters panel data model indicated that the (OLS) estimates are more efficient than the (MG) estimates in all countries. As for the panel data model for random parameters, the results indicated that the estimates of the mean group (MG) method are more efficient than the (FGLS) method in estimating the model parameters for all countries.

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