



ANALYSIS OF EFFICIENT PORTFOLIO BASED ON MODERN PORTFOLIO THEORY

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Article history:	Abstract:
Received: May 14 th 2023 Accepted: June 14 th 2023 Published: July 17 th 2023	Modern portfolio theory is the most basic theory used in forming an optimal investment portfolio. This article aims to provide an overview of modern portfolio theory and its application in effective portfolio formation.
Keywords: Modern portfolio theory, investment analysis, optimal portfolio, The Sharpe ratio, efficiency market, standard deviation of the portfolio	

INTRODUCTION

Modern portfolio theory provides a systematic approach to optimizing the portfolio's risk and expected return ratios, taking into account the relationship between different assets and their diversification.

Portfolio investing is the process of investing funds in a collection of financial assets consisting of a combination of stocks, bonds and other securities. The purpose of the investment portfolio is to create a diversified portfolio of assets aimed at reducing investment risks and maximizing returns to investors.

An investment portfolio is a set of financial assets consisting of a combination of various securities, i.e. shares, bonds, financial derivatives.

Stocks are the most common component of an investment portfolio. The purpose of forming a portfolio of securities is to create a portfolio with a different risk-return profile than individual assets.

LITERATURE REVIEW

The modern portfolio theory was developed by Harry Markowitz, an American economist and Nobel laureate, in the 1950s. Based on the concept of portfolio optimization, he developed mean-variance analysis, which forms the basis of modern portfolio theory. Harry Markowitz, in a study published in 1952 [1], studied the inverse relationship between portfolio diversification, risk, and expected return in the formation of investors' portfolios. He introduced the concept of effective portfolio to science. According to it, portfolios that provide the highest expected return for a given level of risk or the lowest level of risk for a given expected return.

Harry Markowitz in his scientific article showed the mathematical basis of forming an optimal portfolio of 100 securities based on the theory he created. However, finding a mathematical solution to the formation of an optimal portfolio consisting of a large number of assets on the basis of the technologies of the 60 s was a very difficult task. Therefore, in the current risk conditions, when calculating the expected return

from securities, instead of the return from the portfolio, models designed to estimate the expected return from individual assets began to be created. In particular, William F. Sharpe developed the "Capital Asset Pricing Model" in 1964 to estimate the potential return on capital. In addition, in 1966, the scientist [2] developed the Sharpe coefficient, designed to evaluate the efficiency of the investment portfolio. An article detailing the Sharpe ratio [3] was published in 1994 in The Journal of Portfolio Management. According to him, the Sharpe ratio measures the amount of additional return per unit of additional risk.

"Modern Portfolio Theory and Investment Analysis" [4] contains the theoretical foundations of many models, such as the Single Index Model, Multi-Index Models, and the Arbitrage Pricing Theory (APT), which are necessary for the formation of an optimal portfolio, and the mathematical foundations of its practical application.

RESEARCH METHODOLOGY.

An investment portfolio can be formed based on n assets. We calculate the ratio of each security to form the optimal portfolio when our portfolio consists of 2 securities

To estimate the return of an investment portfolio consisting of two assets, we use the following formula.

$$R_p = X_A * R_A + X_B * R_B(1)$$

Here

R_p - expected income from the investment portfolio

X_A - Standard deviation of asset A

R_A - A is the weight of the asset in the portfolio

X_B - B is the standard deviation of the asset

R_B - B is the weight of the asset in the portfolio

Since our investment portfolio consists of two securities, assets A and B together make up 100% of our portfolio, i.e.

$$X_A + X_B = 1 = 100\%$$



We can express this formula as follows

$$X_B = 1 - X_A(2)$$

If we put this formula in (1), the formula of the expected income from the investment portfolio will look like this.

$$R_P = X_A * R_A + (1 - X_A) * R_B$$

The expected return on the portfolio is the simple weighted average of the expected return on the individual securities.

We can find the standard deviation of the portfolio using the following formula

$$\sigma_P = (X_A^2\sigma_A^2 + X_B^2\sigma_B^2 + 2X_AX_B\sigma_{AB})^{1/2}(3)$$

Here

σ_P - standard deviation of portfolio investment

σ_{AB} - Covariance of assets A and B

σ_A^2 - Dispersion of asset A

σ_B^2 - B is the dispersion of the asset

If we add (2) to this formula, our formula will look like this.

$$\sigma_P = (X_A^2\sigma_A^2 + (1 - X_A)^2\sigma_B^2 + 2X_A(1 - X_A)\sigma_{AB})^{1/2}$$

If $\sigma_{AB} = \rho_{AB}\sigma_A\sigma_B$ we take into account that ρ_{AB} - since the correlation between securities A and B is expressed, the formula will look like this.

$$\sigma_P = (X_A^2\sigma_A^2 + (1 - X_A)^2\sigma_B^2 + 2X_A(1 - X_A)\rho_{AB}\sigma_A\sigma_B)^{1/2}$$

The standard deviation of a portfolio is not equal to the arithmetic mean of the standard deviations of each asset in the portfolio.

After finding its standard deviation in the formation of the portfolio, the efficiency of the portfolio in choosing the optimal portfolio is evaluated by the Sharpe coefficient.

Sharpe ratio - investment or **An investment portfolio** was developed by William F. Sharp to evaluate the efficiency of investment income at the level of risk in the market [1].

Sharp coefficient is the angle coefficient between the expected return from the portfolio and the risk-free rate.

$$\text{Sharpe ratio} = \frac{(R_P - R_f)}{\sigma_P}$$

Here

R_P - Expected portfolio return

R_f - Risk-free rate of return

σ_P - Standard deviation of portfolio return

The higher the Sharpe ratio, the higher the investment return relative to the amount of investment

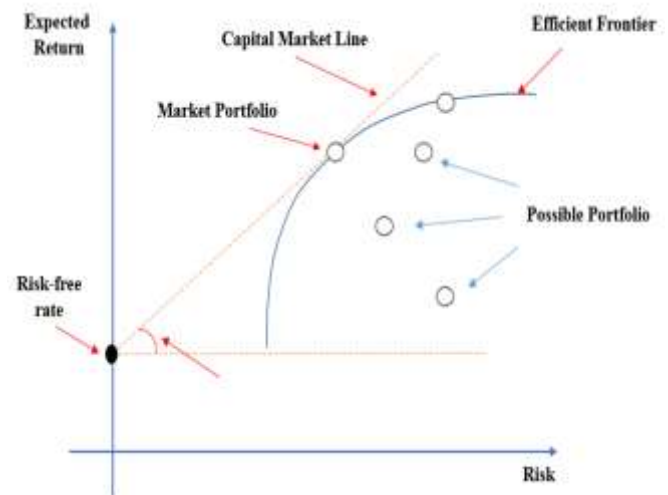
risk, and therefore the higher the efficiency of the investment portfolio. The Sharpe ratio can be used to evaluate the performance of a single stock or an entire portfolio.

The Sharpe ratio is necessary for the investor to estimate the excess expected return for his investment, knowing the existing risk in the market. However, there is no exact limit of the Sharpe ratio in the assessment of the efficiency of the investment portfolio, but many scientists have proposed limits that have a recommendatory character. According to him,

If the Sharpe ratio is < 1, the efficiency of the investment portfolio is low, that is, the investment income does not sufficiently cover the level of accepted risk.

If $1 < \text{Sharpe ratio} < 1.99$, the efficiency of the investment portfolio is interpreted as average or acceptable. This range indicates whether the investment is providing a return commensurate with the risk.

If the Sharpe ratio is > 2, the efficiency of the investment portfolio is considered high. An investment portfolio provides a strong risk-adjusted return, and the higher this ratio, the higher the investment performance.



Graph 1. Risk and Return Graph For n Assets

Sharpe ratio estimates the excess return per unit of risk. When the Sharpe ratio is used to evaluate the efficiency of investments or to compare several portfolios, it is necessary to consider it along with other relevant indicators and comprehensively analyze the risk and return characteristics of investments

ANALYSIS AND RESULTS.

We mentioned that our portfolio consists of 2 stocks. Our portfolio consists of shares of Apple and Microsoft companies. We downloaded the data needed



for our analysis from www.finance.yahoo.com. We have downloaded daily statistics of shares of both companies on the stock exchange for the last 5 years. For the analysis, we need to get not the daily price of

shares, but their daily change. For this, we need to get the logarithm of the daily price of shares of both companies. The purpose of logarithmization is to bring variables into a single unit of measurement.

Table 1
The statistics of changes in shares of Apple and Microsoft companies in the last 5 years

The date	Apple	Microsoft		
20/02/2018	46.23	100.41	Apple	Microsoft
21/02/2018	45,54	98.39	-1.50%	-2.03%
22/02/2018	46.11	99.08	1.23%	0.70%
23/02/2018	46.04	97.54	-0.15%	-1.57%
26/02/2018	46,38	98.63	0.72%	1.11%
27/02/2018	46,28	98.61	-0.21%	-0.02%
28/02/2018	46.79	100.01	1.11%	1.41%
.....				
08/02/2023	183.31	334.29	-0.26%	0.73%
09/02/2023	183.95	337.34	0.35%	0.91%
10/02/2023	186.01	348.10	1.11%	3.14%
13/02/2023	184.92	342.33	-0.59%	-1.67%
14/02/2023	185.01	338.05	0.05%	-1.26%
15/02/2023	183.96	333.56	-0.57%	-1.34%

In this table, 5-year statistics of shares of companies on the stock exchange are given in the 2nd and 3rd categories. In the 4th and 5th columns, the results obtained as a result of logarithmization of the 5-

year share value are returned. The next task is to find the stock's daily mean change, standard deviation, covariance, and risk-free rate needed to determine portfolio performance.

Table 2
General statistical indicators of shares of Apple and Microsoft companies

	General statistics	
Average change	0.110%	0.096%
Standard deviation	2.101%	1.968%
Covariance	0.032%	
Risk free rate	0.008%	

Analyzing this chart, the daily average change of Apple stock is higher than the daily average change of Microsoft stock, so the standard deviation of Apple company is higher, which means that this indicator is 2.101%. we can see. Since the standard deviation also represents the risk associated with the change in stock prices, we can evaluate the standard deviation result of

both companies positively. 0.032% result was obtained from k ovariation of shares. In this table, the risk-free rate is also given. Since the companies selected for analysis are located in the United States, we calculate the 5-year forward change statistics of 1-year government treasury bonds issued by this country, and we take the result as the risk-free rate. Some of the



information needed to find the risk-free rate was downloaded from www.home.treasury.gov

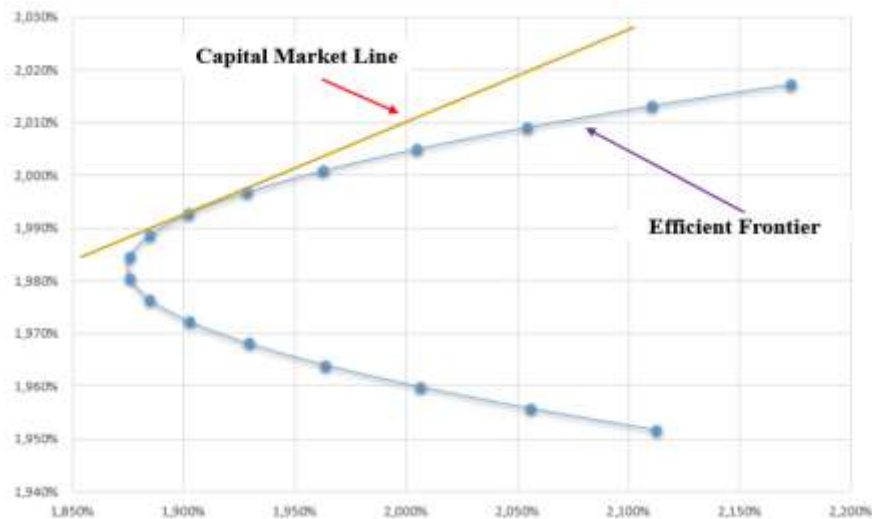
Table 3
Statistics for the formation of an optimal investment portfolio consisting of two assets

Portfolio weight		Portfolio Statistics		
R_A	R_B	R_P	σ_P	
<i>Apple</i>	<i>Microsoft</i>	<i>Return</i>	<i>Standard deviation</i>	<i>Sharpe ratio</i>
130%	-30%	2.017%	2.173%	92.46%
120%	-20%	2.013%	2.111%	95.00%
110%	-10%	2.009%	2.054%	97.40%
100%	0%	2.005%	2.005%	99.60%
90%	10%	2.001%	1.963%	101.53%
80%	20%	1.997%	1.928%	103.13%
70%	30%	1.993%	1.902%	104.33%
60%	40%	1.989%	1.885%	105.08%
50%	50%	1.985%	1.876%	105.36%
40%	60%	1.980%	1.876%	105.13%
30%	70%	1.976%	1.885%	104.41%
20%	80%	1.972%	1.903%	103.22%
10%	90%	1.968%	1.929%	101.59%
0%	100%	1.964%	1.964%	99.59%
-10%	110%	1.960%	2.006%	97.29%
-20%	120%	1.956%	2.056%	94.74%
-30%	130%	1.952%	2.112%	92.01%
Optimal portfolio		1.985%	1.876%	105.36%

This table shows the mathematical result of forming an optimal portfolio consisting of two assets. According to it, in columns 1 and 2 - the weight ratio of each asset in the portfolio is given. The main goal in forming an optimal portfolio is to find the proportion of assets that will bring the highest return from the portfolio. In column 3 (1), the expected return from the portfolio was calculated using the formula. 4 - the standard deviation of the portfolio is calculated, and the smaller this indicator, the lower the risk of the portfolio.

In the last step 5, the Sharpe ratio is calculated, and as we mentioned above, this ratio determines the efficiency of the portfolio, and depending on this indicator, we find the random weight of the assets in the portfolio. This technique serves as a mathematical basis for us to form an optimal portfolio. The highest Sharpe ratio was 105.36% when the asset ratio was 50:50. So, we can conclude that the ratio of two assets should be 50:50 in order for our portfolio to have an optimal return.

We can illustrate this result in a graph as follows.



Graph 2. The combination of assets in the formation of an optimal portfolio

The graph shows the efficient market curve, which incorporates various combinations of assets in the portfolio. It also gives the capital market curve, the intersection of which with the efficient market curve gives us the optimal portfolio.

CONCLUSIONS

In conclusion, in our opinion, in order to increase the volume of trading in the stock market, to assess the internal value and profitability of shares, it is necessary to first of all increase the transparency of the information distributed on the stock market, and to form a wide information base necessary for evaluating shares on stock market sites. In addition, several reasons can be given for the low trading volume in the stock market. These are:

- Low level of financial literacy in both legal entities and individuals. Many large LLCs operating in Uzbekistan prefer expensive bank loans instead of financing their activities by changing their legal status to JSCs and placing their securities on the stock market, that is, they have little awareness of how effective it is to attract financial resources from the stock market. In addition, many individuals do not know how to invest in stocks.

- There are few securities freely traded on the stock market. Today, the number of JSCs in Uzbekistan has increased to 600, and the shares of all of them are not freely traded on stock exchanges. Although the privatization processes are being carried out intensively from year to year, for some reason, this does not lead to a rapid increase in the number of joint-stock companies.

In our opinion, how the above problems will be solved in the coming years and the formation of a

perfect legislative framework regulating the capital market is the main factor determining the development prospects of Uzbekistan's stock markets.

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