World Bulletin of Management and Law (WBML)
Available Online at: https://www.scholarexpress.net
Volume-6, Jaunary-2022
ISSN: 2749-3601

COMPARE BETWEEN ORDINARY LEAST SQUARE AND MAXIMUM LIKELIHOOD METHODS FOR ESTIMATE PARAMETER OF FUZZY SPATIAL LAG MODEL

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Article history:

Received: November 20th 2021
Accepted: December 20th 2021
Published: January 30th 2022

Abstract:

This paper deals with the study about the formulation of Spatial Lag model for independent and dependent fuzzy variables. while the parameters crisp values, with compare between ordinary least square (OLS) and maximum likelihood (MLE) methods to Estimate Parameters by Criteria Root Mean Squares Error (RMSE) and Mean Absolute Percentage Error (MAPE) ,we get results that (OLS) the best from (MLS) for Trapezoidal fuzzy number in the domain traffic accidents for a number of cities in Iraq for the year 2018.and that after converting the Trapezoidal fuzzy number into crisp values by centriod method , calculations the results by Matlab language.

Keywords: Fuzz spatial regression models. Fuzzy Spatial Lag model , centriod method

1- INTRODUCTION

Spatial econometrics is one of the concepts of traditional econometrics, Because it deals with the spatial phenomenon of each variable on the basis of place, these phenomena are specific and known measurements and the errors resulting from them are random variables that can be controlled by studying their behavior [4] , As for fuzzy statistics, it has recently emerged after the emergence of the theory of fuzzy aggregates to be concerned with phenomena whose variables cannot be measured in points, but rather measured in periods, or what is described as uncertain cases or cases with fuzzy data because of its characteristics that make them unclear such as variables that belong in certain proportions to their aggregates. It has no complete affiliation, as well as linguistic variables that cannot be measured numerically, and there are variables that are measured roughly, but in fact they are ambiguous. As a result, fuzzy logic has become applied in many fields. The Artificial Intelligent model, especially in the field of artificial intelligence, is a technique that has a mechanical ability to find solutions to various scientific and applied problems, This is one of the reasons that prompted us to study the fuzzy logic in the general linear spatial regression, we get fuzzy pure spatial autoregressive model for fuzzy trapezoid data by centroid method, and we use least squares to estimated parameters.

2- BASIC CONCEPTS IN FUZZY LOGIC

1-2- Fuzzy Set

It is set whose components have value of belonging, called the degree of membership, which are real numbers within the closed interval [0,1], and the degree membership is expressed as $\mu_A(x)$ that represents the degree of belonging the element from the variable X to The fuzzy set A is written as:

$$A = \big(x\,,\mu_A(x)\big) \quad : \quad \mu_A(x) : x \to [0,1]$$

The memberships change from full or complete to non-membership, or partial membership . **[11] [5]**

2-2- Crisp Set

They are the elements that have a specific characteristic, which takes one of the two values, (1) when the element belongs to the set and (0) when the particular element does not belong to the set .it is called crisp set to distinguish it from the fuzzy set in concepts, let we have a set A known as a function and called the characteristic function as :

$$\chi_A(x) = \begin{bmatrix} 0 & if & x \notin A \\ & & \\ 1 & if & x \in A \end{bmatrix}$$

3- 2-Alph Cat Set $(\alpha - cat\ set)$

Let A fuzzy subset in universal set X , then we define an $(\alpha - \text{cat set})$ of A as:

$$A_{[\alpha]} = \{x \in X : \mu_A \geq \alpha\} \quad , \quad \alpha \in [0,1]$$

4-2- Strong $(\alpha - cat\ set)$

Let A fuzzy subset in universal set X , then we define a strong $(\alpha - \text{cat set})$ of A as:

$$A_{[\alpha^+]} = \{x \in X : \mu_A > \alpha\} \quad , \quad \alpha \in [0,1]$$

5-2- Normalized Fuzzy Set (Core)

A fuzzy subset A in universal set X is called normalized (Core) if :

$$Sup_{x \in X}\mu_A(x) = 1$$

6-2- Convex Fuzzy Set

A fuzzy subset A in universal set X is called convex if :

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$\mu_A(\mathrm{t}) \geq \min\,[\mu_A\,(r)\;,\,\mu_A(s)\,]$ and $t = \lambda r + (1-\lambda)s$
where $r\,,s\;\in R$, $\lambda\;\in[0,1]$.

7-2- Fuzzy number

A fuzzy subset A in universal set X is called fuzzy number if satisfy following condition :

1- convex fuzzy set
2- normalized fuzzy set (maximum membership value is 1)
3- it's membership function is piecewise continuous.
4- It is defined in the real number. **.[6] [7]**

8-2- Triangular fuzzy number

A fuzzy subset A in universal set X is called Triangular fuzzy number that expressed as $A = (a, b, c)$ where $a < b < c$ if has membership functions as:

$$\mu_A(x) = \begin{bmatrix} \dfrac{x-a}{b-a} & if \quad a \leq x \leq b \\ \dfrac{c-x}{c-b} & if \quad b \leq x \leq c \\ 0 & other\;wise \end{bmatrix} \dots\dots\dots\dots\dots (1)$$

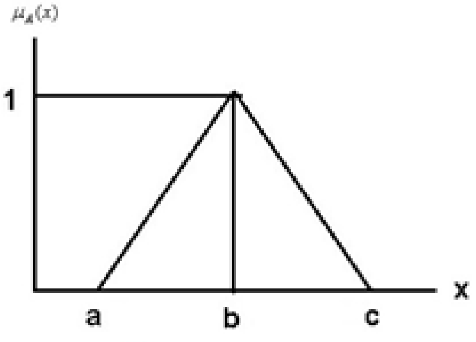
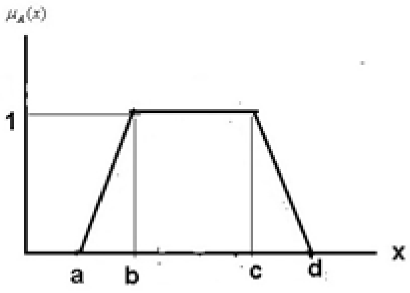


figure (1) Triangular Fuzzy Number

9-2- Trapezoidal fuzzy number

A fuzzy subset A in universal set X is called Trapezoidal fuzzy number that expressed as $A = (a, b, c, d)$ where $a < b < c < d$ if has membership functions as:

$$\mu_A(x) = \begin{bmatrix} \dfrac{x-a}{b-a} & if \quad a \leq x \leq b \\ 1 & if \quad b \leq x \leq c \\ \dfrac{d-x}{d-c} & if \quad c \leq x \leq d \\ 0 & other\;wise \end{bmatrix} \dots\dots\dots\dots\dots (2)$$



Figure(2) Trapezoidal Fuzzy Number

10-2-Convert Fuzzy Number To Crisp Number (Defuzzification)

Let A fuzzy number we can transform A to crisp by centriod method this process is called defuzzification ,the centriod method has the following formula:

$$A_C = \frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx} \dots\dots\dots\dots\dots (3)$$

This method purposed by Sugeno in 1985 is the most commonly used
technique and it is very accurate **. .[10] [12] [3]**

11-2- Convert crisp number to fuzzy number (Fuzzyfication)

The convert process crisp number to fuzzy is called Fuzzyfication , and use membership function in convert which requires have range from zero and one, as shown in the following figure:

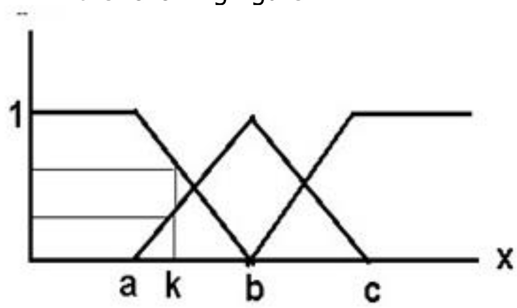


Figure (3) Fuzzyfication

3 - MATHEMATICAL MODEL OF FUZZY MULTIPLE LINEAR REGRESSION

The mathematical model for fuzzy linear regression is defined as:

$$\tilde{y}_i = \beta_0 + \beta_1\tilde{x}_{1i} + \beta_2\tilde{x}_{2i} + \cdots \dots . + \beta_n\tilde{x}_{ni} + \tilde{e}_i \quad , \; i = 1,2, \dots\dots m$$

$$\tilde{Y} = \beta\tilde{X} + \tilde{E} \dots\dots\dots\dots\dots\dots (4)$$

Where
$\tilde{X} = [1, \tilde{x}_{1i}, \tilde{x}_{2i}, \dots\dots\dots .. \tilde{x}_{ni}]$ vector of independent fuzzy variables, expressed as follows : $\tilde{x} = (x_L, x_M, x_R)$ where x_L left side, x_R right side and x_M center value.

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Volume-6, Jaunary-2022
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$\beta = [\beta_0, \beta_1, \beta_2, \dots \dots .. \beta_n]$ vector of parameters of fuzzy regression model are real value.
$\tilde{Y} = [\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \dots \dots .. \tilde{y}_n]$ vector of dependent fuzzy variables, expressed as follows: $\tilde{y} = (y_L, y_M, y_R)$.
$\tilde{E} = [\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \dots \dots .. \tilde{e}_n]$ vector represents the fuzzy errors of the model and written as: $\tilde{e} = (e_L, e_M, e_R)$. **[14]**

1-3 –Parameter Estimation For Fuzzy Regression Models By Centroid Method

In traditional general linear regression

$$Y = \beta X + E \dots \dots \dots \dots \dots . (5)$$

Where
$Y = [y_1, y_2, y_3, \dots \dots .. y_n]^t$
$\beta = [\beta_0, \beta_1, \beta_2, \dots \dots .. \beta_n]^t$
$E = [e_1, e_2, e_3, \dots \dots .. e_n]^t$ where $e \sim N(0, \sigma^2 I)$

$$X = \begin{bmatrix} 1 & x_{11} & x_{12} \dots \dots .. x_{1P} \\ 1 & x_{21} & x_{22} \dots \dots \dots x_{2P} \\ . & . & . \\ . & . & . \\ 1 & x_{n1} & x_{n2} \dots \dots \dots . x_{nP} \end{bmatrix}$$

Then the lest squares estimator of β is

$$\hat{\beta} = (X^t X)^{-1} X^t Y$$

As for fuzzy general linear regression that has model (4) where $(\tilde{x}_{ij}, \tilde{y}_{ij})$, $i = 1,2, \dots \dots n$ and $j = 1,2, \dots \dots \dots . P$ are observational data set of fuzzy input and output and all observations are triangular fuzzy numbers. And $\beta_0, \beta_1, \beta_2, \dots \dots \dots . \beta_n$ are real value , $\tilde{e}$ error terms are also fuzzy number, Let
$\tilde{Y} = [\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \dots \dots .. \tilde{y}_m]^t$
$\tilde{E} = [\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \dots \dots .. \tilde{e}_m]^t$

$$\tilde{X} = \begin{bmatrix} 1 & \tilde{x}_{11} & \tilde{x}_{12} \dots \dots .. \tilde{x}_{1P} \\ 1 & \tilde{x}_{21} & \tilde{x}_{22} \dots \dots \dots \tilde{x}_{2P} \\ . & . & . \\ . & . & . \\ 1 & \tilde{x}_{n1} & \tilde{x}_{n2} \dots \dots \dots . \tilde{x}_{nP} \end{bmatrix}$$

The model in (5) we can written as matrix form :

$$\tilde{Y} = \beta \tilde{X} + \tilde{E} \dots \dots \dots \dots \dots . (6)$$

Since $\tilde{X}_{ij}$, $\tilde{Y}_i$ and $\tilde{E}_i$ are fuzzy triangular number then written as
$\tilde{x} = (x_L, x_M, x_R)$, $\tilde{y} = (y_L, y_M, y_R)$ and $\tilde{e} = (e_L, e_M, e_R)$. the fuzzy data are transformed into crisp data by the centroid method with formal:

$$x_C = \frac{\int x \mu_{\tilde{x}}(x) dx}{\int \mu_{\tilde{x}}(x) dx} = \frac{1}{3}(x_L + x_M + x_R) \dots \dots \dots . \dots \dots . (7)$$

$$y_C = \frac{\int y \mu_{\tilde{y}}(y) dy}{\int \mu_{\tilde{y}}(y) dy} = \frac{1}{3}(y_L + y_M + y_R) \dots \dots \dots \dots . (8)$$

Where x_C and y_C crisp data and $\mu_{\tilde{x}}(x)$, $\mu_{\tilde{y}}(y)$ membership function for x_{ij} and y_{ij} defined in (2.5) and (2.6) Then the estimator for β is:

$$\hat{\beta} = (X^t{}_C X_C)^{-1} X^t{}_C Y_C$$

Where

$$X_C = \begin{bmatrix} 1 & x_{11C} & x_{12C} \dots \dots .. x_{1PC} \\ 1 & x_{21C} & x_{22C} \dots \dots \dots x_{2PC} \\ . & . & . \\ . & . & . \\ 1 & x_{n1C} & x_{n2C} \dots \dots \dots . x_{nPC} \end{bmatrix}$$

$Y_C = [y_{1C}, y_{2C}, y_3, \dots \dots .. y_n]^t$
[2] [8]

4 - FUZZY SPATIAL LINEAR REGRESSION (FUZZY SPATIAL LAG MODEL)

The form of general spatial model that contain both spatially lagged and error structure spatial correlation , As shown in the following formula :

$$Y = \lambda WY + Z\beta + u \quad , \quad |\lambda| < 1 \dots \dots \dots \dots \dots (9)$$
$$u = \rho W u + e \quad , \quad |\rho| < 1$$

Where
$Z = [X \quad WX]$ and $\beta = [\beta_1 \quad \beta_2]$
Y : vector $(n \times 1)$ of depend variable.
X: a matrix of non-stochastic regression.
W: weight matrix.
$e/X = i.i.d. N(0, \sigma^2_{\epsilon n} I_n)$
If we assuming that $\lambda \neq 0$ and $\rho = 0$ in model (9) we get to spatial lag model (SLM) or spatial autoregressive model (SARM) , or mixed regressive model , where insert independent variable spatial lag as once observes variable WY and spatial lag parameter on response variable λ it is describe the strength of spatial response, the mathematical formula for this model is :

$$Y = \lambda WY + Z\beta + e \dots \dots \dots \dots . (10)$$
$$e \sim N(0, \sigma^2_{\epsilon n} I_n)$$

This model (10) we can expression as fuzzy model as

$$Y_C = \lambda W Y_C + Z_C \beta + e_C \dots \dots \dots \dots (11)$$
$$e_C \sim N(0, \sigma^2_{\epsilon n} I_n)$$

Where :
Y_C : vector $(n \times 1)$ is centroid of trapezoidal fuzzy number are depend variable .
I : Identity matrix $(n \times n)$.
λ : is parameter of spatial regression .
W: spatial weights matrix$(n \times n)$.
Z_C : matrix $(n \times k)$ is centroid of trapezoidal fuzzy number are observation variables .
β: vector $(n \times 1)$ of parameter require estimate him .
e_C : vector $(n \times 1)$ is centroid of trapezoidal fuzzy number are spatial random error. **[5] [9]**

1-4 - Ordinary Least Square (OLS)for (FSLM)or(FSARM)

The fuzzy observation depended variables that fuzzy spatial lagged model (11) are associated with the term fuzzy error ,where it is breaches one of the hypotheses of (OLS) ,then the results of this method to estimation are biased and inconsistent because $W y_C$ with error e_C are not independent , to derive

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Volume-6, Jaunary-2022
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estimation formulas by ordinary least square we use the following steps :

$$Y_C = \lambda W Y_C + Z_C\beta + e_C$$

$$e_C = Y_C - \lambda W Y_C - Z_C\beta$$

$$e'_C e_C = Y'_C Y_C - 2Y'_C Z_C\beta - 2\lambda Y'_C W Y_C + 2\lambda Y'_C W' Z_C\beta + \beta' Z'_C Z_C \beta + \lambda^2 Y'_C W' W Y_C$$

We derive $(e'_C e_C)$ to β and equal to zero we get:

$$\Rightarrow \hat{\beta} = (Z'_C Z_C)^{-1} Z'_C Y_C - \lambda (Z'_C Z_C)^{-1} Z'_C W Y_C \quad \dots\dots\dots\dots (12)$$

$$\Rightarrow \hat{\beta} = b_0 - \lambda b_1$$

and again derive $(e'_C e_C)$ to λ and equal to zero we get:

$$\Rightarrow \hat{\lambda} = [Y'_C W' W Y_C - b'_1 Z'_C W Y_C]^{-1} [Y'_C W Y_C - b'_0 Z'_C W Y_C] \dots. \dots\dots\dots (13)$$

And the variance is

$$\sigma^2 = \frac{\left(Y_C - \hat{\lambda} W Y_C - Z_C\hat{\beta}\right)' \left(Y_C - \hat{\lambda} W Y_C - Z_C\hat{\beta}\right)}{n-k} \dots\dots\dots\dots (14)$$

[1] [13].

2-4 - Maximum Likelihood Estimation (MLE) for (FSLM) or (FSARM)

Since the Maximum likelihood estimation considered one of the most important ways because it gives the best estimate for parameter from among several possible estimates so the researcher applied this method to estimate parameter fuzzy spatial autoregressive model (11) as:

$$\Rightarrow Y_C - \lambda W Y_C = Z_C\beta + e_C$$

Thus, the log likelihood function for Y of the fuzzy spatial autoregressive model is obtained by adding the term $Ln|I - \lambda W|$ to the log likelihood function of the standard regression model

$$L(\beta, \lambda, \sigma^2 / Y_C, Z_C) = \frac{1}{(2\pi\sigma^2)^{n/2}} |I - \lambda W| \exp\left[-\frac{1}{2\sigma^2} e'e\right] \dots\dots\dots (15)$$

$$Ln(L) = -\frac{n}{2} Ln(2\pi) - \frac{n}{2} Ln\sigma^2 + |I - \lambda W| - \frac{1}{2\sigma^2} e'e \dots\dots\dots\dots\dots (16)$$

Where

$$e_C = Y_C - \lambda W Y_C - Z_C\beta \dots\dots\dots\dots.. (17)$$

Putting equation (17) in equation (16) we get :

$$Ln(L) = -\frac{n}{2} Ln(2\pi) - \frac{n}{2} Ln\sigma^2 + |I - \lambda W| - -\frac{1}{2\sigma^2} (Y_C - \lambda W Y_C - Z_C\beta)'(Y_C - \lambda W Y_C - Z_C\beta) \dots\dots\dots\dots (18)$$

On account of this correction the Maximum Likelihood Estimation (MLE) estimates will differ from the Ordinary Least Square (OLS) estimates. They coincide for $\lambda = 0$ where the fuzzy spatial autoregressive model approaches the standard regression model.

we get the derivative for β ,σ^2 in log of likelihood and equal to zero we get :

$$\beta_{MLE} = (Z'_C Z_C)^{-1} Z'_C A Y_C$$

Where

$$A = (I - \lambda W)$$

$$\beta_{MLE} = (Z'_C Z_C)^{-1} Z'_C (I - \lambda W) Y_C = (Z'_C Z_C)^{-1} Z'_C Y_C - \lambda (Z'_C Z_C)^{-1} Z'_C W Y_C$$

$$b_O = (Z'_C Z_C)^{-1} Z'_C Y_C \quad and \quad b_L = (Z'_C Z_C)^{-1} Z'_C W Y_C$$

$$\beta_{MLE} = b_O - \lambda b_L \text{............}(19)$$

$$e_C = Y_C - \lambda W Y_C - Z_C \beta_{MLE} = Y_C - \lambda W Y_C - Z_C (b_O - \lambda b_L) = Y_C - Z_C b_O - \lambda (W Y_C - Z_C b_C)$$

$$e_{OC} = Y_C - Z_C b_O$$

$$e_{LC} = W Y_C - Z_C b_L$$

$$e_C = e_{OC} - \lambda e_{LC} \dots\dots\dots.. (20)$$

According to our condition the error variance can be estimated by:

$$\sigma^2_{MLE} = \frac{(e_{OC} - \lambda e_{LC})'(e_{OC} - \lambda e_{LC})}{n} \dots\dots\dots\dots\dots (21)$$

Where :

b_O: vector of the regression parameter Y_C for Z_C

b_L: vector of regression parameter WY_C for Z_C

λ : parameter of spatial regression model .

e_O : vector of another regression model Y_C for Z_C

e_L : vector of another regression model WY_C for Z_C

by formulation for the calculation the determinant $|I - \lambda W|$ as below:

$$|I - \lambda W| = \prod_{i=1}^{n} (I - \lambda w_i)$$

$$Ln|I - \lambda W| = \sum_{i=1}^{n} Ln(I - \lambda w_i)$$

After get the Eigen values for weights matrix, the solution can be using the (Nonlinear optimization) method, then putting the parameters in (concentrated likelihood function) as follows:

$$L_C = -\frac{n}{2} Ln\left[\frac{1}{n}(e_{OC} - \lambda e_{LC})'(e_{OC} - \lambda e_{LC})\right] + Ln|I - \lambda W| \dots\dots. (22)$$

By using the iterative method for concentrated likelihood function ,we can obtained λ value. [4]

5 - SPATIAL WEIGHTS MATRIX (ROOK CONTIGUITY)

It is a square matrix which it elements positive value, and it is not necessary to be symmetric , and it is based on geographic arrangement of the observations, or contiguity , i.e. the relations among location with other location in one row of the matrix and the diagonal elements in the matrix are equal to zero ,the Spatial weights matrix by Rook contiguity is equal to 1 if the two areas (locations) neighbor by limited and have relation between the two areas (locations) in any side, and it is equal to 0 otherwise. This matrix used in applications more than the others. [2] [10]

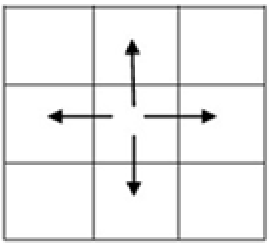

Figure (1) Shows the Rook weight matrix

6 - MORAN TEST FOR SPATIAL REGRESSION

It is a general measure depends on the general linear model GLM and uses for autocorrelation coefficient (called the Moran coefficient because Moran is the name of the Scientist that find the test)

the Moran formula is:

where we using row – standardized where sum of row equal to 1 in this case $(n = S_0)$ that is work to simplify the Moran's formula as follows :

$$I = \frac{(e'we)}{(e'e)} \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

To know if the value of the Moran coefficient it is Indicator Statistics in certain degree of confidence we must use moran (Z) test with Hypotheses:

$H_0 = \lambda = 0 \quad , \ \theta = 0 \qquad there\ is\ no\ spatial\ dependence$

$H_1 : at\ lest\ one\ of\ \ \lambda \neq 0 \quad or \quad \theta \neq 0\ \ there\ is\ spatial\ dependence$

$$Z = \frac{I - E(I)}{\sqrt{V(I)}} \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

$$E(I) = \frac{n(tr(MW)}{S_0(n-k)}$$

$$V(I) = \frac{tr(MWMW') + (tr(MW))^2 \ + (tr(MW))^2}{(n-k)(n-k+1)} (\frac{n}{S_0})^2 - (E(I))^2$$

Where :

$M = I - X(X'X)^{-1}X'$ Idempotent Matrix that is $(n \times n)$ and symmetric.

tr : Sum diagonal element .

k : Number of explanatory variables.

The calculated value Z is compared with value of Z table for $(^{\alpha}/_2)$, and where Moran test is significant that is mean relation between geographic location that refers to use spatial regression model and not enough general linear model (GLM) and we have spatial dependency. **[13]**

7-COMPARISON CRITERIA FOR CHOOSING THE BEST MODEL

The method of selecting a particular model from among a many of models is an important aspect of data analysis as it leads us to choose the best model , where the use of certain statistical criteria are as described below:

1-7- Root Mean Squares Error (RMSE)

It is the square root of the sum of the squares errors divided by (n-k-1) and is calculated for all models, and the model which the value of the square root of the mean squares is smaller errors is the best model, and the general formula for it is:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n-k-1}} \ldots\ldots\ldots\ldots (25)$$

2-7- Mean Absolute Percentage Error (MAPE)

It is the sum of dividing the absolute value of the error by the real value divided by the number of observations (n). The general formula for it is:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{(y_i - \hat{y}_i)^2}{y_i}\right| \ldots\ldots\ldots\ldots\ldots (26)$$

And the smaller value is the best model.**[4]**

8- PRACTICAL PART

1-8 – Estimate initial value

In this part of the paper, we used, real data these data represent the number of deaths $\tilde{Y}_i$ resulting from traffic accidents in Iraq for six governorates (Baghdad, Anbar, Diyala, Salah al-Din, Kirkuk, Nineveh) by 38 observations. And this traffic accidents as Crash accidents $\tilde{X}_1$ and Overturn accidents $\tilde{X}_2$ and Run over accidents $\tilde{X}_3$,this data represent trapezoidal fuzzy number and has membership function, in this paper explains given idea about dealing with such data, in this peper transform fuzzy data into Crisp data, by the formula :

$$X_C = \frac{1}{4}(X_L + X_{M1} + X_{M2} + X_R) \ \ , \ \ Y_C = \frac{1}{4}(Y_L + Y_{M1} + Y_{M2} + Y_R)$$

So the fuzzy multiple regression formula for this data is

$$Y_{Ci} = \beta_0 + \beta_1 X_{C1i} + \beta_2 X_{C2i} + \beta_3 X_{C3i}$$

Where

Y_{Ci}: the number deaths in traffic accidents (D.A)

X_{C1i} : Crash accidents (C.A)

X_{C2i} Overturn accidents (O.A)

X_{C3i} : Run over accidents (R.A)

We get β_i by ordinary least square

Table (1) estimated initial value of beta

World Bulletin of Management and Law (WBML)
Available Online at: https://www.scholarexpress.net
Volume-6, Jaunary-2022
ISSN: 2749-3601

Model	Constant	C.A	O.A	R.A
β_i	7.3078	0.0544	0.7951	0.2653

2-8- Moran Test

The initial values that estimated are used in the Moran test, which we obtained the following result:

Table (2) result moran test

I	**Z(I)**
-0.0372	0.7878

Since the value of $Z(I)$ is less than the tabular value of Z in terms of (α/2), then we accept the null hypothesis, which states that the data is spatially dispersed and randomly.

3-8- Estimate parameter FSARM By Using (OLS) and (MLE)

To estimate the parameters of the Fuzzy Spatial Lag model , we calculated spatial matrix weights between locations with 38×38 by (Rook contiguity), and we are have model is as follows:

$$Y_C = \lambda W Y_C + Z_C \beta + e_C$$

And after estimating the parameter β and λ by Ordinary Least Square method as in formula (12) and (13) we get :

Table (3) result estimating the parameter β and λ by OLS

β	15.9304	0.0043	0.6194	0.3492	-0.0321	0.2436	0.0362
λ	-0.0911						

And we estimating the parameter β and λ by Maximum Likelihood method as in formula (19) and (22) we get :

Table (3) result estimating the parameter β and λ by OLS

β	17.1208	0.0208	0.5448	0.3420	-0.0395	0.1752	0.0244
λ	- 0.0455						

4-8- Calculate Different Criteria by Using (SAR) Model

After estimate β , λ for SAR model by ordinary least square and maximum likelihood methods, and using rook matrix, we must use criteria for finding the best method that estimate the model.

Table (4) Calculate Different Criteria by Using (SAR) Model

Method	**RMSE**	**MAPE**
OLS	14.6093	1.4955
MLE	14.7019	1.5101

The results showed in the table above as follows :

1 - RMSE: Is a measure used for differences between the value predicted by a model or an estimator and in this paper the value actually observed in RMSE is the smaller value in OLS method and this means that the method of OLS is the best method to estimate the SAR model.

2 - MAPE: is measure of prediction accuracy of a forecasting method in statistics, in this measure the smaller value is in OLS method than LME method, this mean that the OLS is the best method to estimate SAR model.

9- RECOMMENDATIONS

1- Using maximum likelihood and ordinary least square method to compare between another model as Fuzzy Pure Spatial Autoregressive Model , Fuzzy Lagged Independent Variable Model and Fuzzy Spatial Error Model
2- Fuzzyfication the data into triangular fuzzy number and comparing it with the trapezoidal fuzzy number for showing a difference between them in the estimate.
3- There are other methods than the centriod method in dealing with fuzzy data such as alpha cat $(\boldsymbol{\alpha - cat})$.
4- Study and application real data on other fuzzy spatial models

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World Bulletin of Management and Law (WBML)
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Volume-6, Jaunary-2022
ISSN: 2749-3601

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Table (4) centroid data and results error

	Location Cities	Y_{Ci}	X_{C1i}	X_{C2i}	X_{C3i}	$\widehat{Y}_{Ci}$ $by\ OLS$	e	$\widehat{Y}_{Ci}$ $by\ MLE$	e
1	Mosul	52	6	1	68	38.2191	13.7809	37.3078	14.6922
2	AL-Hamdaniya	16	26	5	20	24.0230	-8.0230	23.4866	-7.4866
3	Telkaif	16	6	1	4	15.8728	0.1272	15.4191	0.5809
4	Sinjar	4	6	1	4	15.8728	-11.8728	15.4191	-11.4191
5	Tel afar	16	6	5	4	18.3505	-2.3505	17.5982	-1.5982
6	AL-Hatra	16	6	1	4	18.9088	-2.9088	15.9700	0.0300
7	Maqmoor	16	6	1	4	15.8728	0.1272	15.4191	0.5809
8	Kirkuk	40	46	13	52	35.0639	4.9361	38.5128	1.4872
9	Daquq	28	26	21	20	28.7601	-0.7601	31.5104	-3.5104
10	Debes	28	6	13	4	18.1322	9.8678	21.2638	6.7362
11	Tikrit	16	66	1	52	19.1687	-3.1687	20.5350	-4.5350
12	Tuz kurmato	4	6	5	4	4.6286	-0.6286	5.0489	-1.0489
13	Samara	16	6	5	4	4.6286	11.3714	5.0489	10.9511
14	Baled	4	6	1	4	2.1510	1.8490	2.8698	1.1302
15	AL- Dor	4	6	1	4	2.1510	1.8490	2.8698	1.1302
16	AL- shargat	4	6	5	4	4.6286	-0.6286	5.0489	-1.0489
17	Baquba	88	186	13	132	71.6216	16.3784	70.0222	17.9778
18	AL- meqdadia	16	46	9	4	23.8579	7.8579	21.1938	-5.1938
19	AL-Kalus	76	66	17	20	34.4859	41.5141	31.4403	44.5597
20	Kanaqeen	4	46	13	4	26.3356	-22.3356	23.3728	19.3728
21	Baladrouz	16	6	9	4	23.6859	-7.6859	20.3613	-4.3613
22	Al-Rasafa	64	106	17	132	59.3779	4.6221	59.312	4.6875
23	AL-aadamia	16	46	13	52	28.6878	-12.6878	28.4198	-12.4198
24	AL-sader2	16	26	9	36	20.5375	-4.5375	20.3523	-4.3523
25	AL-sader1	4	26	5	52	23.6464	-19.6464	23.6454	-19.6454
26	AL-Karek	40	126	17	116	53.7913	-13.7913	53.8403	-13.8403
27	AL-Kademia	4	66	9	68	31.8827	-27.8827	32.1291	-28.1291

World Bulletin of Management and Law (WBML)
Available Online at: https://www.scholarexpress.net
Volume-6, Jaunary-2022
ISSN: 2749-3601

28	AL-Mahmoodia	40	26	5	20	12.4733	27.5267	12.7011	27.2989
29	Abu-griab	16	6	1	4	4.3230	11.6770	4.6336	11.3664
30	AL-Taremia	4	6	1	4	4.3230	0.3230	4.6336	-0.6336
31	AL-Madayn	40	6	1	20	9.9096	30.0904	10.1058	29.8942
32	AL-Rumadi	16	146	5	20	20.6299	-4.6299	23.4049	-7.4049
33	Heet	4	6	1	4	11.9206	-7.9206	12.6319	-8.6319
34	AL-Faloga	4	26	9	4	16.9620	-12.9620	17.4063	-13.4063
35	Anah	4	6	1	4	11.9206	-7.9206	12.6319	-8.6319
36	Haditha	4	6	5	4	14.3983	10.3983	14.8110	-10.8110
27	AL- Rutba	16	6	5	4	14.3983	1.6017	14.8110	1.1890
38	AL-qaem	28	6	5	4	14.3983	13.6017	14.8110	13.1890