



## INTEGRATION BETWEEN THE SCIENCE OF MATHEMATICS AND THE SCIENCE OF MATHEMATICS TEACHING METHODS

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Article history:	Abstract:
<b>Received:</b> 11 <sup>th</sup> November 2022 <b>Accepted:</b> 14 <sup>th</sup> December 2022 <b>Published:</b> 30 <sup>th</sup> January 2023	This article discusses the integration between the science of mathematics and the science of methods of teaching mathematics in order to improve the quality of higher mathematical education in the context of its humanization; the authors propose to integrate mathematical and methodological training.
<b>Keywords:</b> Elementary mathematics, methodical training, methodical orientation of teaching, methodical skills, task	

The main task of the modern school is "... revealing the abilities of each student, educating a personality ready for life in a high-tech, competitive world. training should contribute to personal growth so that graduates can independently set and achieve serious goals, be able to respond to different life situations" [1]. Obviously, these words of D. A. Medvedev have full meaning for higher education. The intellectual potential of society, which serves as the most important factor in the country's economic and social development, its political independence, and a factor in its survival, is determined by the quality of higher education. The task of improving the quality of higher education is paramount for any country in the world.

In the light of the foregoing, the task of higher educational institutions includes not only the professional training of future specialists, but also the development of their universal values, the formation of creative activity, the ability to make decisions independently in difficult situations, etc. Deepening and expanding integrative ties between various components of the professional training of students at the level of goal-setting, content, methods, forms and means of education. In this article, in the context of the humanization of education, we consider the possibilities of integrating the mathematical and methodological training of students of a pedagogical university in teaching them elementary mathematics.

The training course we are considering occupies an important place in the training of future teachers of mathematics, serving as a kind of bridge between the cycles of mathematical and methodological disciplines. If the courses of mathematical analysis, algebra and geometry provide a scientific justification for all the concepts introduced in school mathematics, and the process of solving problems in these disciplines is aimed primarily at working out certain aspects of the concept being studied, then in practical classes in elementary mathematics, the main attention is paid to solving tasks.

At the same time, students not only master the methods of solving the problem, but also strive to reveal the process of finding a solution, choosing appropriate methods of reasoning, modeling school learning situations, which, in turn, contributes to the formation of the foundations of the methodological skills of the future teacher of mathematics. Therefore, due to the specifics of the professional training of future teachers, teaching elementary mathematics to students should be methodically directed.

Under the methodological orientation of teaching students elementary mathematics in a pedagogical university, we mean the purposeful formation of methodological activity, which, in turn, is defined as an activity that implements the functions of the theory and methodology of teaching mathematics: methodological, prognostic, explanatory, descriptive, systematizing, educational, heuristic, aesthetic, practical, normative and evaluative. Each function corresponds to one or another aspect of methodological activity. According to T.B. Sattorov, the formation of methodological skills adequate to each of these aspects is an indicator of the quality of methodological training of students [2].

An analysis of the component composition of methodological skills identified in various aspects of the methodological activity of students of a pedagogical university shows that in the formation of a methodological orientation in teaching elementary mathematics, the greatest attention should be paid to the prognostic, aesthetic and heuristic aspects of the teacher's activity. This is due to the fact that the totality of the actions corresponding to them: to put forward and analyze hypotheses, focus on finding new ways of solving and proving, developing and solving creative problems, revealing the aesthetics of solving problems, etc. - are included in the content of other aspects of methodological activity, and therefore, fully contribute to the methodological preparation of students. In addition, recently the aesthetic education of students,



the formation of research skills, and creative activity have been recognized as topical tasks of teaching mathematics at school. The solution of these problems presupposes the mastery of future teachers with skills adequate to the prognostic, heuristic and aesthetic aspects of methodological activity.

Currently, according to educational standards [1], the content of elementary mathematics includes the study of arithmetic-algebraic and geometric topics. Hence, in the implementation of the methodological orientation of teaching this discipline to students of a pedagogical university, two main lines can be distinguished:

1) the formation of methodological skills that implement the functions of methodological science,

when students study the arithmetic and algebraic part of elementary mathematics;

2) the formation of methodological skills that implement the functions of methodological science, when students study the geometric part of elementary mathematics [3].

The second line corresponds to the systematic study by students of a pedagogical university of the theory and methodology of teaching mathematics in the III-IV courses, while their study of the arithmetic-algebraic line of elementary mathematics takes place in the I-III courses. Therefore, in teaching elementary mathematics to students, two stages of implementing the methodological orientation can be distinguished, each of which corresponds to a certain level of formation of the above methodological skills (Table 1).

**Table 1**

**The relationship between the levels of formed methodological skills and the stages of implementation of the methodological orientation of teaching elementary mathematics**

Levels	Stages
Low	Formation of methodological skills in the study of the arithmetic-algebraic part of elementary mathematics in the period preceding the systematic study of the course of theory and methods of teaching mathematics (pre-methodological stage)
Middle	
High	Formation of methodological skills in the study of the geometric part of elementary mathematics during the period of systematic study of the course of theory and methods of teaching mathematics (methodological stage)

As you can see, at the stage of students studying the arithmetic-algebraic part of elementary mathematics, their methodological activity turns out to be formed at a low level. They lack the skills to put forward and analyze hypotheses, select heuristics, identify original solutions, etc., requiring the guidance of a teacher. The foregoing is also true for the stage of studying the geometric part of elementary mathematics by students in the fourth year. But since students systematically studied the course of theory and methods of teaching mathematics in parallel, the formation of their methodological skills in elementary mathematics classes can be raised to a higher level. At the same time, in addition to solving individual problems, blocks of enlarged problems have great efficiency.

A block of enlarged tasks is a construction of several tasks combined into one whole on the basis of the principle of common activity for their solution. The solution of each subsequent task in the block enlarges

the solution of any of the block tasks preceding it by performing new actions that complement its solution.

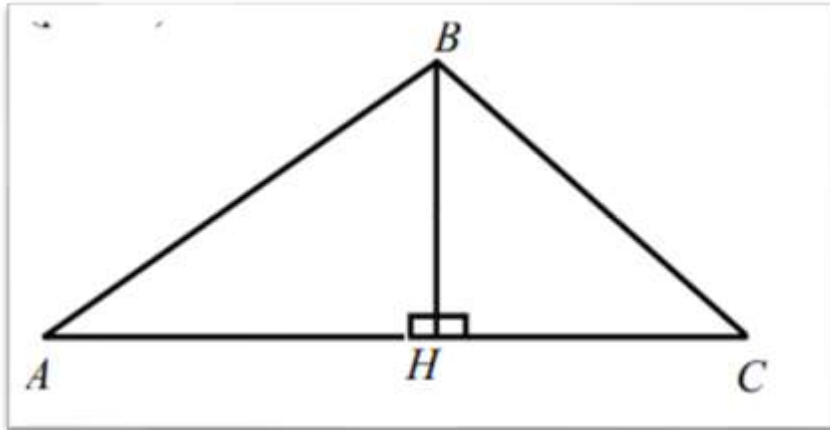
The methods for forming blocks of enlarged tasks are the replacement of a task requirement with some new requirement; extension of the task drawing; task reversal; replacing the condition of the problem with some new condition [3].

Such blocks can be easily formed, in particular, by developing the theme of the problem being solved at the final stage of working with it. This stage can serve as a testing ground for the development of students' creative initiative, heuristic and independent thinking, the search for a more rational and aesthetically attractive way of solving, etc., since its implementation, among other things, involves the formulation of new tasks, including aggregating actions to solve the original problem.

Let us demonstrate what has been said on the example of the tasks included in block 1.1-1.3.

**1.1.** In a triangle ABC, height BH = h, sides AB = a, BC = b. Calculate the area of the triangle (Fig. 1)

**Fig. 1**



Mathematical and methodological aspects of solving this problem are presented in Table 2.

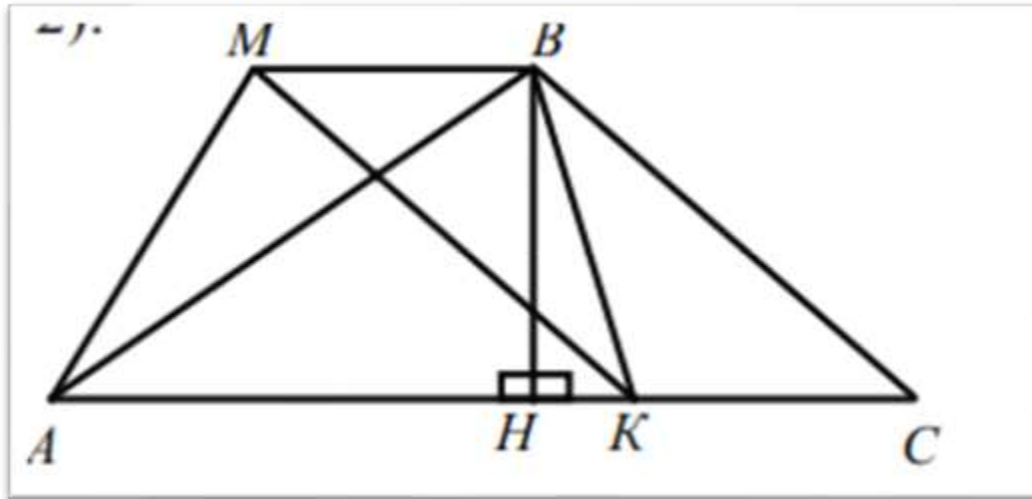
**Table 2.**

**Mathematical and methodical analysis of the solution of the problem 1.1.**

Math skills	Methodical skills
The area of a triangle can be found using the Heron formula (1) or using the formulas $S = \frac{1}{2} h AC$ (2), $S = \frac{1}{2} ab \sin \angle B$ (3)	Obtaining advanced information about an object
In formulas (1) and (3), we do not know two quantities each (the side AC, the perimeter of the triangle; $\angle B$ , $\sin \angle B$ , respectively). In formula (2), only one side of the AC is unknown. Therefore, in order to find the area $\Delta ABC$ required in the problem; it is more expedient to find its side AC	Choosing a Forecasting Basis Putting forward a hypothesis
AC = AN + NC. Therefore, in order to find AC, it is necessary: 1. Find AN from $\Delta ABN$ using the Pythagorean theorem; 2. Find the NC from the $\Delta BNC$ using the Pythagorean theorem; 3. Find AC as the sum of AN and NC	Planning for the upcoming activity

To develop the solution of the specified task 1.1, students can be offered task 1.2.

**1.2.** In triangle ABC, height BH = h, sides AB = a, BC = b. An arbitrary point K is taken on the segment NS. On the line BM parallel to AC, a point M is taken so that MB = KC. Calculate the ratio of the area of the triangle ABC and the trapezoid AMBK (Fig. 2)



**Fig. 2**

When solving problem 1.2, students perform the same actions as when solving problem 1.1. In addition, they need to calculate the area of the trapezoid AMBK using the formula  $S = \frac{1}{2} h (MB + AK)$  and find the required area ratio. In other words, the solution to problem 1.2 really extends the solution to problem 1.1 by performing new actions.

At the final stage of working with task 1.2, the teacher can focus students' attention not only on the method of the trapezoid AMBK described in the task based on the triangle ABC, but also on the possible reverse transition from this trapezoid to this triangle

**Table 3**  
**Mathematical activity of students in solving problems 1.1-1.3**

Actions adequate to solve the problem			Mathematical knowledge, skills, abilities
1.1	1.2	1.3	
		By carrying out a parallel transfer of the side MK to the vector $\vec{MB}$ , we get the side BC;	The ability to see the image of an object with a parallel transfer of the plane
$AN = \sqrt{a^2 - h^2}$ $NC = \sqrt{b^2 - h^2}$ $AC = AN + NC$ $S_{\Delta ABC} = \frac{1}{2} h AC$			Knowledge of the Pythagorean theorem Ability to find the area of a triangle
	$S_{\Delta ABC} =$ $= \frac{1}{2} h (AK + MB) =$ $= \frac{1}{2} h AC = S_{\Delta ABC}$		Ability to find the area of a trapezoid
	It follows from the previous paragraph that the required area ratio is equal to		

Heuristics can be a continuation of the work:

- 1) "to find the area of a certain figure, it is enough to find the area of a figure of equal size to it";
- 2) "if you construct a triangle with a side equal to the sum of the bases of some trapezoid, then the

area of this triangle will be equal to the area of this trapezoid".

To consolidate these heuristics, it is logical for students to propose problem 1.3, which enlarges the solution of problem 1.2.



**1.3.** In the trapezoid AMBK, the diagonals are  $AB = a$ ,  $MK = b$ . The height of the trapezoid is  $h$ . Calculate the angle between the diagonals of the trapezoid (see Fig. 2).

The solution of the problem is shown in Table.

3. If the methodological side of solving problem 1.1 includes the methodological skills formed by students, which mainly implement the prognostic function of methodological science (the prognostic aspect of their methodological activity), then the solution of problem 1.2, in addition to the prognostic function of methodological science, also implements the heuristic function. The simplicity and unexpectedness of solving Problem 1.3 by means of a parallel transfer of the diagonal of the MK trapezoid to the  $\overline{MB}$  vector contributes to the implementation of the aesthetic

aspect of methodological activity (an unexpected relationship between the original trapezoid and a triangle of equal size, about which nothing was originally said in the task).

Thus, we see that the activity of transforming a separate task in elementary mathematics in the analysis of its solution allows students to form various methodological skills, which in combination contribute to the implementation of the methodological orientation of the training course, thereby carrying out their methodological training at the proper level. The consistent relationship between such skills, implemented when solving blocks of enlarged problems, can be similar to the one that exists between the components of students' mathematical activity formed in this case (Table 4)

**Table 4**  
**Methodological activity of students in solving problems 1.1–1.3**

Methodological skills adequate to solve the problem			The aspect of methodological activity
1.1	1.2	1.3	
Obtaining advanced information about the object; Selection of the basis of forecasting; Putting forward a hypothesis; Planning for future activities;			Prognostic
Assimilation of special (private) heuristics			Heuristic
Mathematical construction transformation; Orientation to the search for a new, more perfect and original way of solving			Aesthetic

University practice shows that the mathematical training of students in solving blocks of enlarged problems is also carried out at a fundamental level, ensuring their assimilation of holistic, systemic and integrated knowledge. As can be seen, an important role is played by the integration of methodological and mathematical training of future specialists at all possible levels; teaching aids (which are the same tasks of elementary mathematics and their blocks); teaching methods (analysis of the decisions made, search for rational methods for solving problems, establishing dependencies between the components of the problem, etc.); forms of education (use of ready-made tasks in the classroom, organization of independent problem solving by students, etc.).

Such work allows students to form both mathematical skills in solving problems and methodological skills in teaching schoolchildren to solve such problems. At the same time, students easily join research activities, develop self-control and self-

education skills, productive distribution of study time, etc., which, together with the above, contributes to the formation of a highly intelligent technological personality of the future teacher in modern conditions.

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