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THE COMPARISON OF DIFFERENT THRESHOLD RULES IN ESTIMATING THE WAVELET REGRESSION FUNCTION

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Abstract:

The applications of wavelet regression began to enter the field of statistics as a powerful tool in the field of data smoothing. Considering the wavelet method in statistical estimates is one of the very powerful methods that have been spread by means of wavelet shrinkage estimators (Estimators (Wavelet Shrinkage Donoho and Johnstone (1994) and Donoho et al. (1995) estimators have been introduced. Non-linear wavelet in non-parametric regression The wavelet reduction technique is one of the best techniques used in estimating the non-parametric regression function, but it is affected in the method of selecting the appropriate rules and threshold values. Semi-smooth and the second improved threshold In addition to taking the values of Visus hrink and sure shrink threshold values, using three test functions and sample sizes (64, 128, 256, 512) and different noise ratios. The soft thresholds smi rule is followed by the improved method M2 using the visu threshold value. It usually breaks down with the extreme difficulty of providing a suitable threshold value for the wavelet threshold coefficients at all desired levels. This prompted the researchers to find functions and threshold values that suit that problem to obtain efficient wavelet estimations using deflated wavelet regression and different threshold rules.

Keywords: wavelet regression, Threshold Value، Thresholding Rules ,Wavelet Coefficients

INTRODUCTION

Since the beginning of the nineties of the last century, applications of wavelet transformation began to enter the field of statistics as a powerful tool in the field of data smoothing. Considering the wavelet method in statistical estimates is one of the very powerful methods that have been spread by the Estimators Wavelet Shrinkage (where wavelet shrinkage is a method or method to remove Signal distortion is based on the idea of performing a Thresholding of the resulting wavelet coefficients By applying a Wavelet Transformation, the goal is to retrieve an unknown function, for example g, based on the noise-polluted data samples. Noise reduction techniques are a very effective and simple way to find structure in data sets without imposing a parametric regression model, very general assumptions are made about g such that it belongs to a particular class of functions Donoho and Johnstone (1994), Donoho et al. (1995) have introduced wavelet estimators Non-linearity in nonparametric regression by thresholding typically amounts to a per-term evaluation of coefficient estimates in the empirical wavelet expansion of the non-parametric function known. if the estimate of the modulus is large enough in absolute terms - that is, if it exceeds a predetermined threshold - the corresponding term is kept in the experimental wavelet expansion (or reduced to zero by an amount equal to the minimum); Otherwise it was deleted. Therefore, the use of a general threshold usually disintegrates with great difficulty in providing an appropriate threshold value for the wavelet threshold coefficients at all desired levels. This prompted the researchers to find functions and threshold values that suit that problem to obtain efficient wavelet estimations using deflated wavelet regression and different threshold rules.

2 - Wavelet Regression

The concept of wavelet regression or wavelet contraction is considered one of the modern methods for analyzing data and knowing the relationships between its variables. The process of converting data from the time domain to the wavelength domain to estimate an unknown function f(xi), which requires equal distances between points 〖 x〗_i and if the sample size is Daedician n = 〖2 〗^J, let us have minimum observations (, yn....= (y1 yi is given by the following formula:[1],[2]

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yi = f(xi)+εi ……..(1)
and that (x_i)=i/n and the aim is to estimate the unknown function f(xi) which is a nonparametric function, and xi ∈[0,1] and 〖ε〗_i represents the white noise () and it is often assumed that it is independent and symmetrical in distribution (i.i.d) is normally distributed N(0,1), or the estimation can be done by matrix method.
…….(2) ε+ f= y
Since y=(y_1…..y_n), f=(f_1….f_n) and) ε_n….. ε=(ε_1, the wavelet coefficients w are then calculated by applying the discrete wavelet transform, where W is a matrix wavelet transform
w=W y ……(3)
(y_i)̂=W^(-1)w ....(4)
Donoho et al., suggested a threshold-dependent estimation method in the wavelet field and that the thresholding process or wavelet contraction is the main process responsible for noise reduction which depends on the selection of the threshold and the thresholding method. All noise reduction algorithms first find an optimal threshold value. The threshold value and the threshold method are chosen, then the threshold function or the contraction method is chosen. , in wavelet regression the selection of the threshold value is a critical issue. Too large a value reduces too many coefficients resulting in excessive homogeneity. Conversely, a very small threshold value allows many coefficients to be included in the reconstruction, giving a fluctuating estimate that results in heterogeneity. However, the correct choice of threshold can be considered as a delicate balance of these principles and a great deal of research effort is spent on methods for selecting the limit value of wavelet contraction. Most of the early wavelet contraction techniques relied on Mallat's hierarchical algorithm (1989) to calculate the discrete wavelet transform (DWT) through three main steps: [1],[2]
1- The observations are converted to the wavelet field by applying a discrete wavelet transform (DWT). The result is a series of wavelet coefficients di,i=1…n
2 - The wavelet coefficients found in the first step are modified by using a “soft” or “hard” threshold rule to determine the value of the coefficients through wavelet shrinking as shown in Figure (1).
3- The coefficients are inversely transformed by returning to the signal space by taking the inverse of the discontinuous wavelet transform (IDWT) to get the estimated function f̂ .

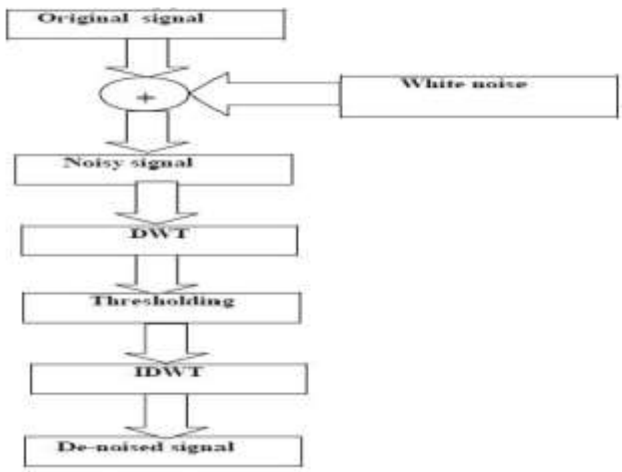

Figure (1) Shows Wavelet Contraction [3]

3- Function Thresholding

Wavelet Thresholding The wavelet threshold is considered one of the basics of wavelet transformation, whether it is discontinuous or continuous, and the threshold technique is a modern non-linear method used to reduce the wavelet noise. Using what is known as wavelet shrinking The wavelet shrinking process was introduced in the year (1995) by “Donho”, which operates on a wavelet modulus one at a time In its simplest form, each modulus is the threshold by comparison with the threshold, if the modulus is smaller than the threshold, set to zero; Otherwise it is kept or modified. There are several types of frequently used threshold functions such as, Hard Thresholding, . and semisoft Thresholdig and Improved Thresholding (2), as shown below: [4]

4- Hard Thresholding

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This method was proposed by Dohono and it represents a linear function, the coefficients below the threshold τ are set to zero while the other coefficients remain unchangedAs shown in equation (5), this method does not change the local properties of the signal, but because of the discontinuity, it leads to Specific oscillation in the reconstruction of the original signal, that is, it is a discontinuous function, and the variance is greater for the estimated function, while the amount of bias and the average error squares are as little as possible and are expressed in the following formula: [6],[7],[5]

〖Th〗_Hard ={█(Y if |Y| >τ@0 if|Y|≤τ)┤ ....(5)

〖 and Th〗_Hard is a representation of the estimated wavelet coefficients, (Y) is a representation of the noisy wavelet coefficients, τ) (it is the threshold, the hard threshold may look good, the hard threshold does not work even with some algorithms sometimes, it may exceed the pure noise coefficients Minimum and appear as annoying "light signals" in the output. Since

Improved Thresholding(2))M2) 6-

This method was suggested by the researchers. Jin, M., Wang, C. The basic idea of removing noise from the wavelet threshold is to perform wavelet decomposition on the scrambled signal to determine the wavelet basis and decomposition layers, and to address the shortcomings of these functions a new improved threshold function is proposed. For this optimized threshold function to distinguish the useful

the hard threshold function is not continuous at the starting point, so there are fluctuations in the recovery of the original signal. Compared to the hard threshold function, [7],[5]

5- Threshold Semisoft Thresholding

This method was suggested by Lu Jing-yi. And others, where the continuity of the soft threshold function is much better, but it still has a constant deviation. Therefore, in order to overcome its shortcomings, among the soft and hard threshold functions, a value of T is taken between 0 and 1. which is an adjustment factor of the function even if the result of a semi-soft threshold function is between them, the value of T is constant for that, there is still Constant bias, so the soft and hard threshold algorithms are compromised by the literature and expressed in the following formula:[6]

〖Th〗_Semisoft ={█(sgn(Y)(|Y|-T τ), if |Y| >τ@0 , if|Y|≤τ)┤ ....(6)

signal from the noise signal more effectively, the constant value of the partial wavelet coefficient compression is used, as well as to improve the stability and quality of the signal more effective de-noising, in addition, there is no uncertainty factor in the improved threshold function, which ensures Noise removal stability. It can be formulated according to the following equation: [8]

$$Th_{Improved(2)} = \begin{cases} sgn(Y)\{|Y| - |Y|.e^{\tau-|Y|}\}, & if\ |Y| > \tau \\ 0 & ,\ if |Y| \leq \tau \end{cases} \quad ......(7)$$

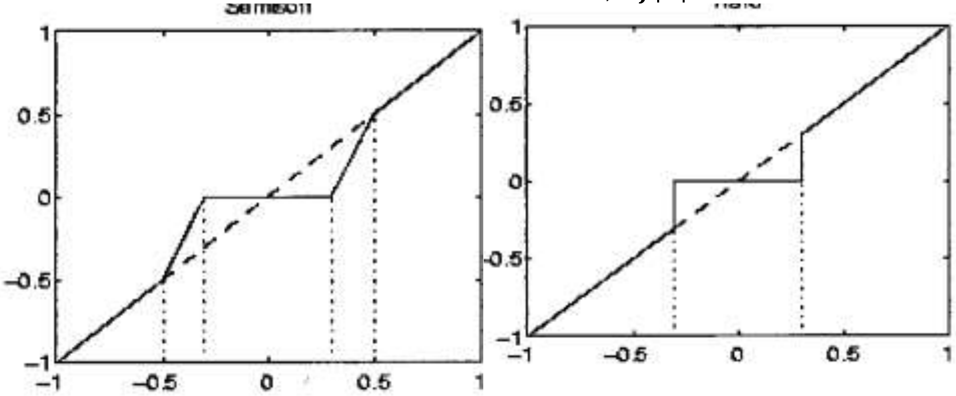

Figure 2 shows the Hard Threshold Function and the Semisoft . Threshold Function

7- Threshold value

There are various ways to choose the threshold value τ, which is an important parameter in the wavelet reduction algorithm to reduce signal noise and is very important and necessary in the wavelet conversion process. If the threshold value is too small, there will still be a lot of noise and the estimator will be oscillating, and if the threshold value is too large, some important features of the signal may be filtered, for this reason the appropriate selection of the

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threshold value has a role in the accuracy of the estimation because the wavelet coefficients pass through the threshold limit, Therefore, optimization of the threshold value is an important criterion for obtaining a minimum MSE. There are many methods presented by (Jonston & Donoho) to determine the threshold value, among these methods:[1],[4]

8- ((Sure Thresholding

This method was introduced by Donoho and Johnstone, which is achieved by the principle of Stein Unbiased Estimation (SURE) Estimation (Stein Unbiased Risk) for each j-wave level, which is indicated at the threshold-dependent level, and the amount of this bias is expressed by the following formula :[1]

)= N-2 ∑_(K=1)^N▒ 〖I(|djk|≤τ_j)〗 + ∑_(k=1)^N▒ 〖min⁡ 〖(|djk|≤τ_j)〗 ^2 〗 . ...(8)
d_(j,k),SURE(τ_j

djk represents the wavelet coefficients, which are orthogonal as they result from the wavelet transformation, which is orthogonal, and the final formula for calculating the Sure Thresholding value is as in the following formula:

τ_(j,sure)= 〖argmin〗 _(0≤τ≤√(2log(N))) SUR(τ_(j,djk)) ....(9)

where τ is the value that underestimates Stein's unbiased risk estimator. Sure Shrink reduces the mean squared error, and it also adjusts for smoothness, which means that if any unknown function includes sudden changes or boundaries in the image, the reconstructed image has the same as well.

9- Visushrink Thresholding)[9]

This method is considered an improvement of the comprehensive threshold method, as it addresses the weakness of this method through its good performance even with an increase in the sample size, as it gives a more homogeneous and preparatory estimate. For n, which leads to a loss of many wavelet coefficients with noise, and therefore the threshold does not perform well at interruptions in the signal, and Visu Shrink does not deal with reducing the mean square error. This method can be illustrated according to the following formula:

σ_n √(2 log⁡(n))⁡ =τ_Visushrink)10).......

〖 σ〗 _n is the standard deviation of the noise level, which can be found through the following relationship:

10-Test Function

:[10]*(Doppler function)*

$\sin\{2\pi(1+\varepsilon)\},\ \varepsilon = f_1(x) = \{x(1-x)\}^{\frac{1}{2}}$ $0.05,$ $\dots\dots(11)$

:[10]*(Heavisinefunction)*

$f_2(x)$= 4sin4πx-sin(x-0.3)-sin(0.72-x)k ....(12)

:[10] *(Blocksfunction)*

$f_3(x)$= $\sum h_j$k(x-x_j), $k(x) = \{1+sgn(x)\}/2$ ....(13)

x_j =(0.1,0.13,0.15,0.23,0.25,0.40,0.44,0.65,0.76,0.78, 0.81)

h_j=(4,-5,-4,5,-4.2,2.1,4.3,-3.1,2.1,-4.2)

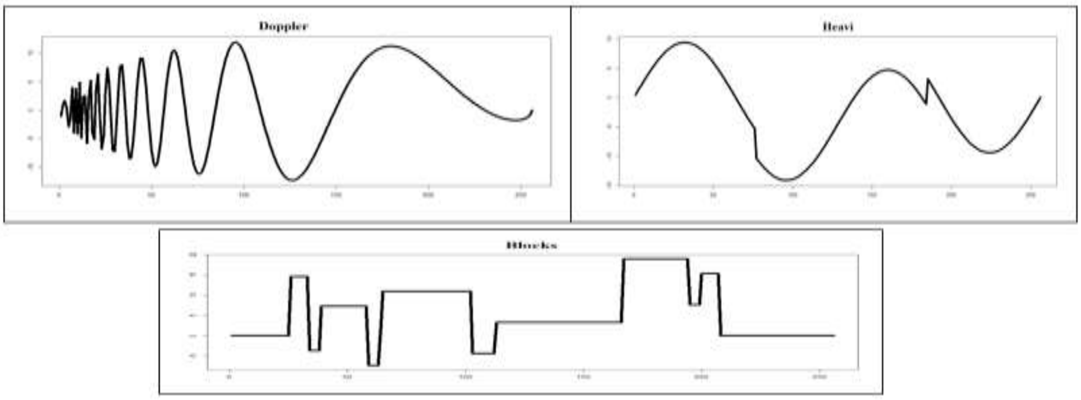

Figure (3) Test functions

(Simulation)

Simulation experiments were used for the purpose of comparing the methods used in estimating the nonparametric regression function in the presence of a correlation in errors of the type (AR(1)) through the use of three test functions and four sample sizes (64,128,256,512) and distortion ratios) SNR = 5) and (SNR=10) and (Hard Thresholding Rule) and (Smi soft Thresholding Rule) and (M2) Improved Tthresholding Rule2 were used. In addition to using two types of threshold values, namely (Sure Threshold) and (Visurink Threshold), the programming language (R 1.3.4) was used in order to generate random variables

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to build simulation models and to compare the methods described in the theoretical side.

CONCLUSIONS:

First: the test function (Doppler), which is shown in formula (11) and figure (3)

1. In general, and according to the different sample sizes and noise levels, we note the superiority of the estimation methods using the visu threshold value and soft thresholds smi, followed by the improved M2 method using the visu threshold value.
2. In general, and with different sample sizes and noise levels, we note the superiority of the estimation methods using the value of the visu threshold, according to the different threshold rules over the sure threshold value.
3. In general, we notice a decrease in the value of MSE with an increase in the sample size, except for the sample size n = 128 and for all methods and with different levels of confusion.
4. The performance of both the method in which a hard threshld rule was used and the second improved method was degraded according to the different sample sizes and noise ratios.
5.

Second: The test function (Heavisin), which is shown in formula (12) and figure (3)

1. In general, and according to the different sample sizes and noise levels, we note the superiority of the estimation methods using the visu threshold value and the soft threshold smi rule, followed by the improved threshold method M2 using the visu threshold value.
2. In general, and with different sample sizes and noise levels, we note the superiority of the estimation methods using the value of the visu threshold, according to the different threshold rules over the sure threshold value.
3. In general, we notice the convergence of the performance of the methods (fabricating the threshold rules) according to the different percentages of disturbance, with a slight fluctuation in the priority according to the different percentages of disturbance.
4. In general, we notice a decrease in the value of MSE with an increase in the sample size, except for the sample size n = 128 and for all methods and with different levels of confusion.

Third: The test function (Blocks), which is shown in formula (13) and figure (3)

1. In general, and according to the different sample sizes and noise levels, we note the superiority of the estimation methods using the visu threshold value and the soft thresholds smi, followed by the hard threshold using the visu threshold value.
2. In general and the difference in sample sizes, we note the superiority of the estimation methods using the threshold value of visu and for all threshold rules at snr = 10
3. In general, and with different sample sizes and noise levels, we note the superiority of the estimation methods using the value of the visu threshold according to the different threshold rules over the sure threshold value.
4. In general, we notice a decrease in the value of MSE with the increase in the sample size, except for the sample size n=128, according to the different threshold rules used, except for the soft threshold method, and the sure threshold value is at the sample size n=256.

Table (1) shows the MASE standard for comparing estimates of the fuzzy Doppler function for sample sizes n=256, n=512, n=128, n=64, and SNR=5

		SNR=5	
	visuSS	visuH	visuM2
64	0.015281391	0.025385795	0.024976028
128	0.015444371	0.025519515	0.025214769
256	0.013649744	0.023050255	0.022535880
512	0.013906215	0.024064499	0.023644305
			SNR=5
	sureSS	sureH	sureM2
64	0.048138453	0.048166493	0.051465243
128	0.052400388	0.052176322	0.049981311

256	0.047563567	0.048295582	0.047453802
512	0.049136034	0.049354812	0.049353882

Table No. (2) shows the MASE standard for comparing estimates of the fuzzy Doppler function for sample sizes n=256, n=512, n=128, n=64, and SNR=10

	SNR=10		
	visuSS	**visuH**	**visuM2**
64	0.008091564	0.013090107	0.022256652
128	0.009219536	0.014963310	0.024423094
256	0.00866952	0.01408972	0.02376652
512	0.007644609	0.012841216	0.021772550
			SNR=10
	sureSS	**sureH**	**sureM2**
64	0.047845049	0.048373100	0.047143832
128	0.049471782	0.049654842	0.050109263
256	0.04858452	0.04879521	0.04857544
512	0.045524300	0.045619892	0.047703694

Table No. (3) shows the MASE criterion for comparing estimates of the Heavisin noise function for sample sizes n=256, n=512, n=128, n=64, and SNR=5

	SNR=5		
	visuSS	**visuH**	**visuM2**
64	0.01616042	0.02638335	0.02554995
128	0.014964049	0.024845241	0.024252560
256	0.01738421	0.02773101	0.02685937

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512	0.01747883	0.02805687	0.02717921

			SNR=5
	sureSS	**sureH**	**sureM2**
64	0.05075757	0.05163425	0.05155199
128	0.051136725	0.051117127	0.050306758
256	0.05301243	0.05336220	0.05308874
512	0.05200169	0.05298663	0.05249794

Table No. (4) shows the MASE criterion for comparing estimates of the Heavisin noise function for sample sizes n=256, n=512, n=128, n=64, and SNR=10

		SNR=10	
	visuSS	**visuH**	**visuM2**
64	0.01762605	0.02808614	0.02735299
128	0.01706571	0.02747606	0.02716265
256	0.01862900	0.02926064	0.02837626
512	0.01672167	0.02656659	0.02590007
			SNR=10
	sureSS	**sureH**	**sureM2**
64	0.05245220	0.05311509	0.05270105
128	0.05144485	0.05265030	0.05311391
256	0.05380816	0.05410877	0.05275088
512	0.05145047	0.05094459	0.05119800

Table No. (5) shows the MASE criterion for comparing estimates of the noise block function for sample sizes n=256, n=512, n=128, n=64, and SNR=5

		SNR=5	
	visuSS	**visuH**	**visuM2**
64	0.04179863	0.05619947	0.05382346
128	0.04302877	0.05792920	0.05494087
256	0.04276670	0.05767663	0.05496527
512	0.04116388	0.05545738	0.05324673
			SNR=5
	sureSS	**sureH**	**sureM2**
64	0.07958583	0.08065839	0.07973684
128	0.08078343	0.08246401	0.08160770
256	0.08066399	0.08164933	0.08016086

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512	0.07894982	0.08049101	0.07730232

Table No. (6) shows the MASE criterion for comparing estimates of the noise block function for sample sizes n=256, n=512, n=128, n=64, and SNR=10

	SNR=10		
	visuSS	**visuH**	**visuM2**
64	0.04237049	0.05724394	0.05451861
128	0.04262823	0.05689512	0.05469935
256	0.04147842	0.05447124	0.05264007
512	0.013097409	0.02275185	0.022305009

			SNR=10
	sureSS	**sureH**	**sureM2**
64	0.07786991	0.0784896	0.08150134
128	0.07821845	0.08029136	0.07699004
256	0.07909752	0.07896077	0.07855923
512	0.04219745	0.05707252	0.05386647

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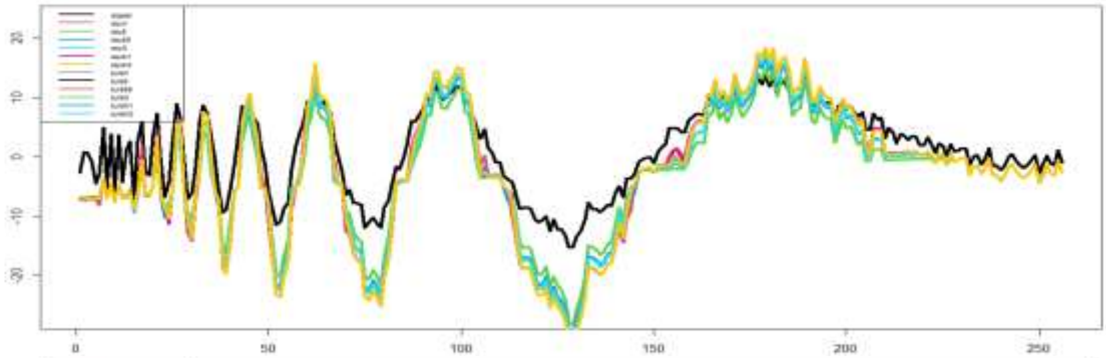

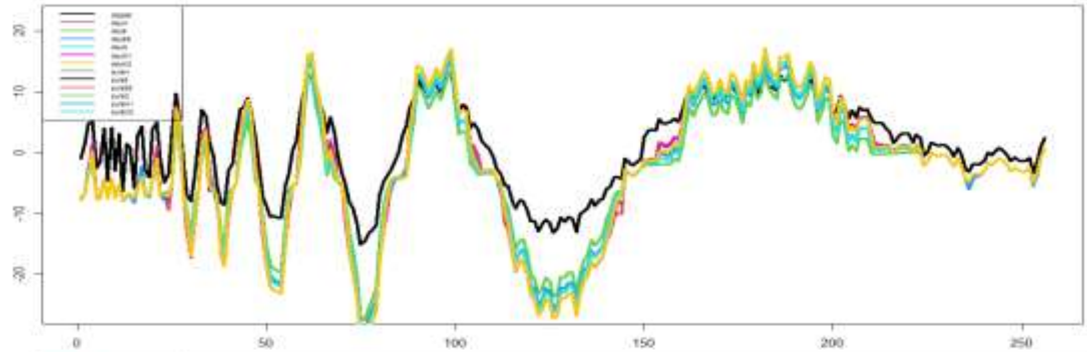

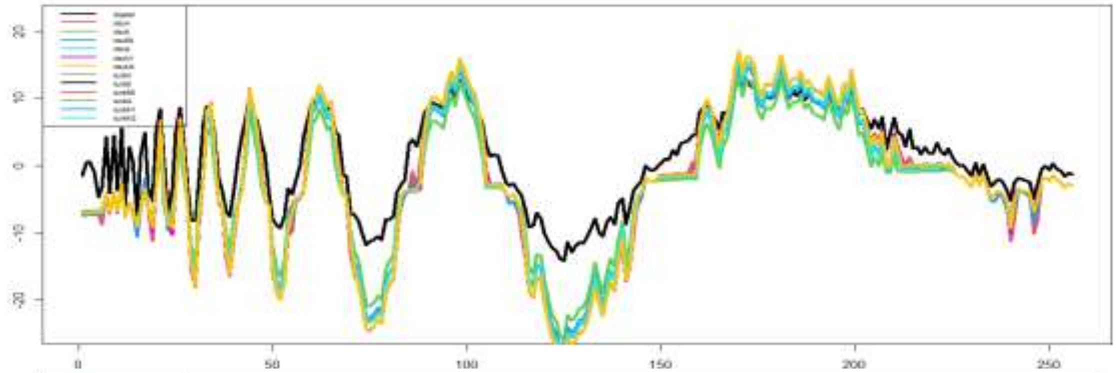

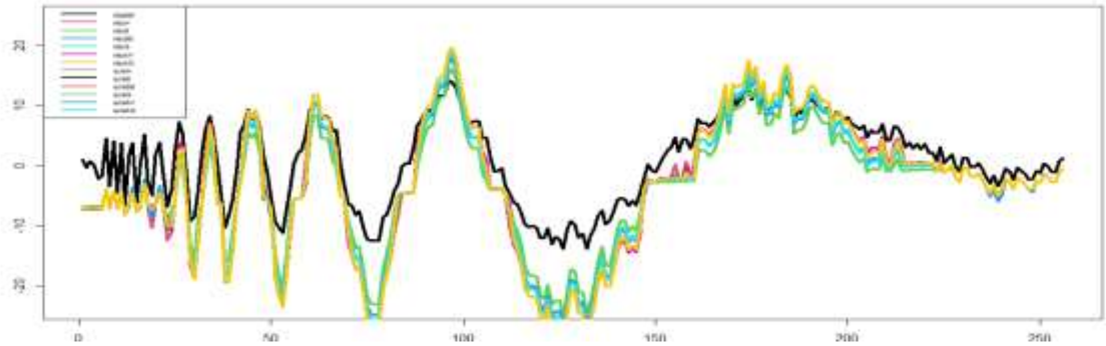

Figure (4) shows the real and estimated values of the dependent variable Y using the dopler function and sample size

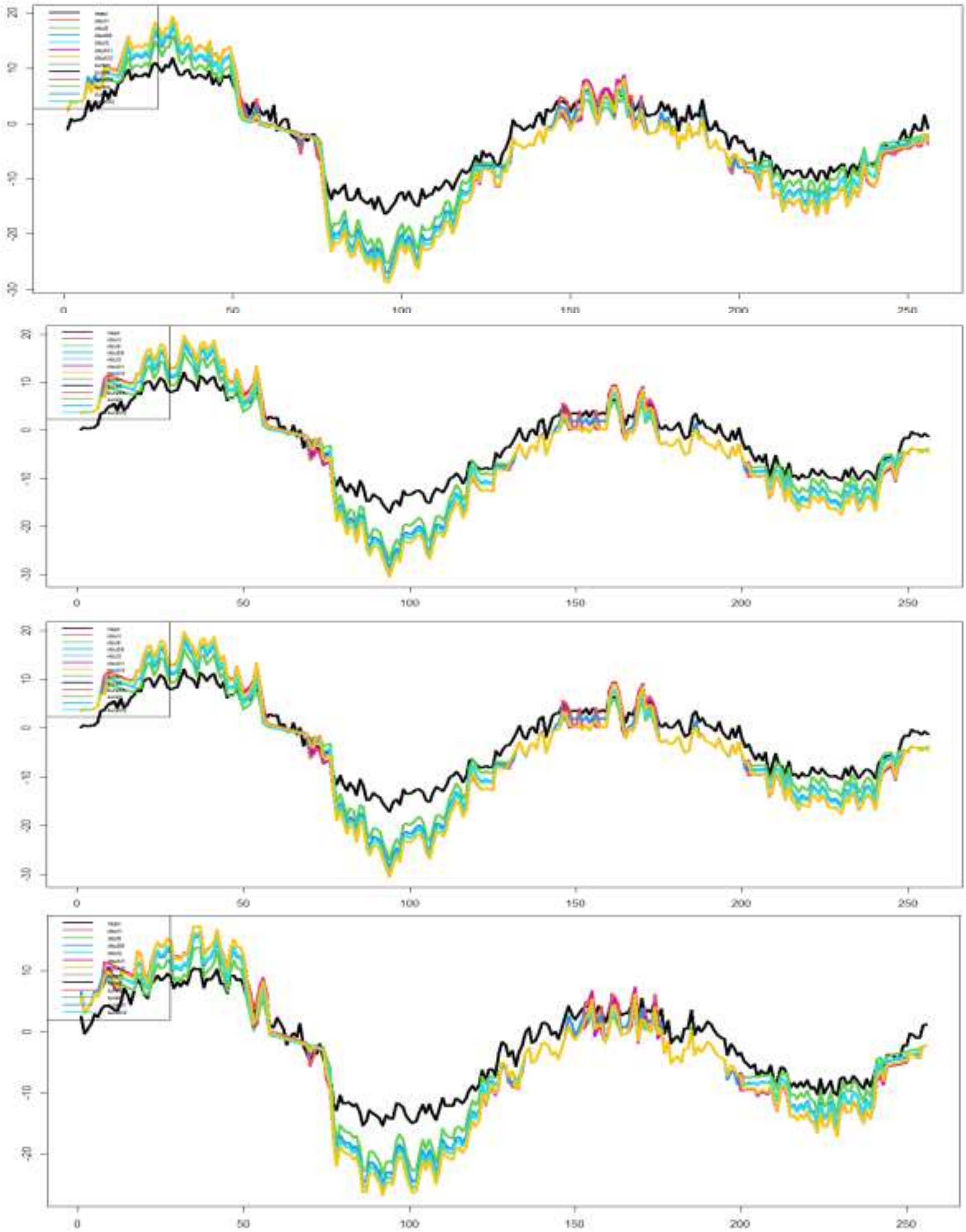

Figure (5) shows the real and estimated values of the dependent variable Y using the heavi function and sample size .(64, 128,256,512), respectively

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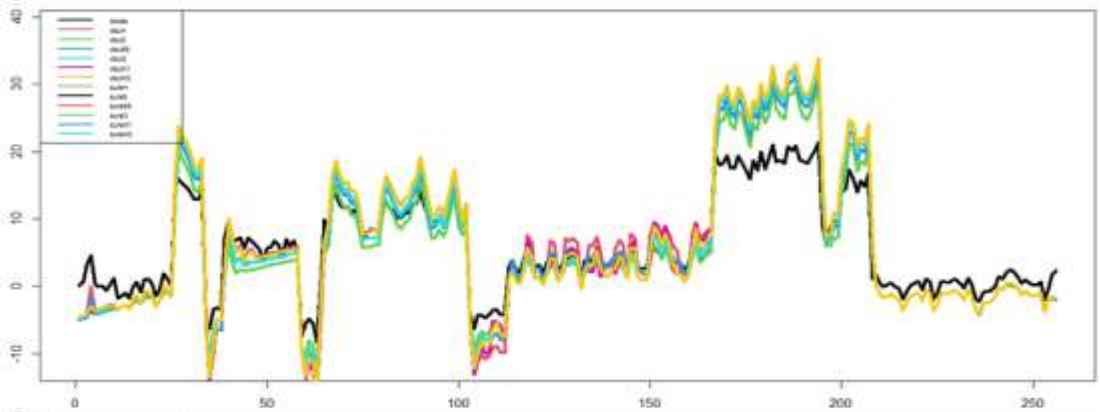

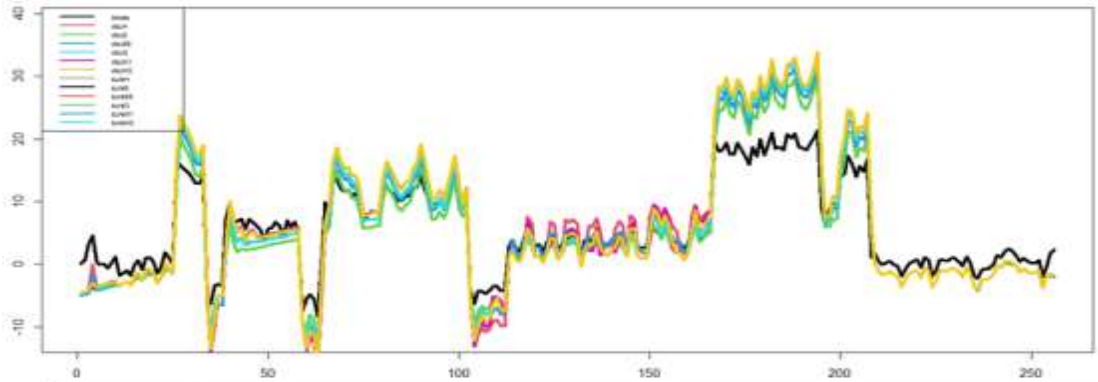

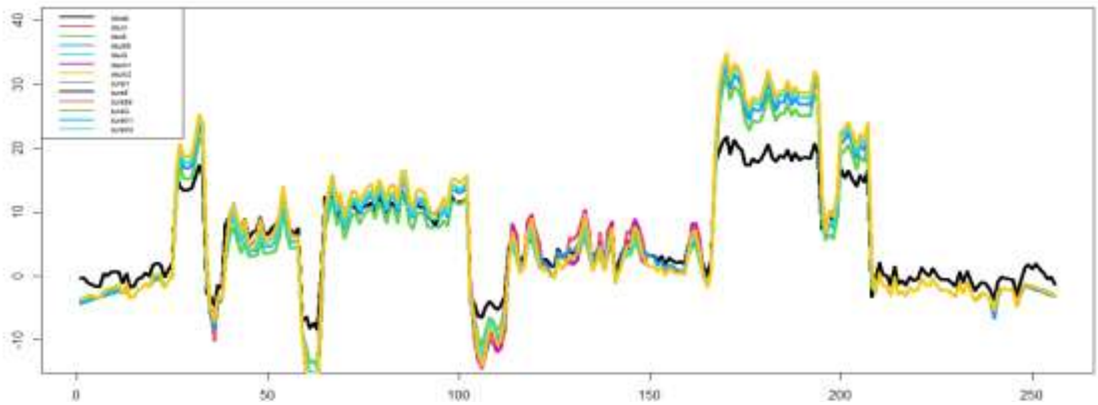

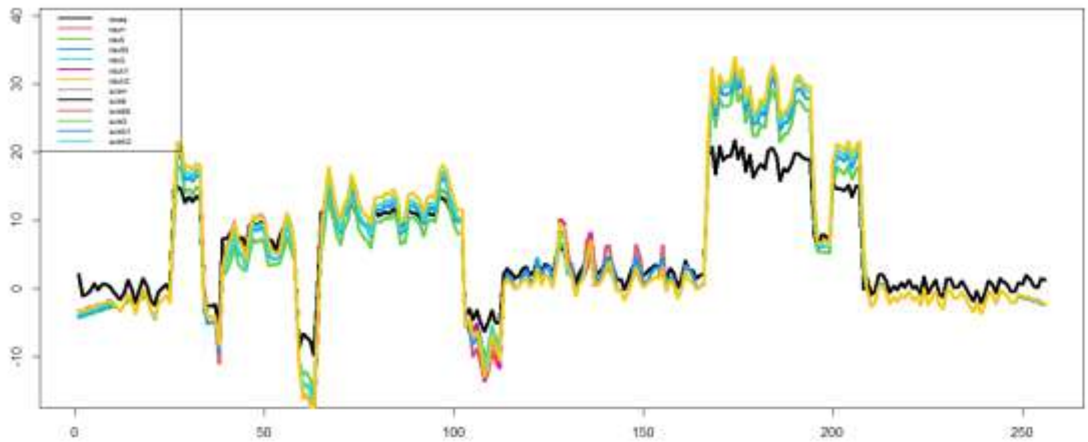

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Figure (6) shows the real and estimated values of the dependent variable Y using the block function and sample size (128,256,512), respectively.

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