



THE ROBUST LOGISTIC REGRESSION MODEL ESTIMATION FOR PATIENTS INFECTED WITH COVID-19 VIRUS

Namareq Qassem Hussain ¹

Noor Amer Harb Albazoon ²

Zahraa Ibrahim Abd Abbas Al-Jubouri ³

¹ Department of Statistics, Faculty of Administration and Economic, Kerbala University, Iraq

Email: Namareq.q@uokerbala.edu.iq

² Department of Statistics, Faculty of Administration and Economic, Kerbala University, Iraq

Email: noor.aamer.@s.uokerbala.edu.iq

² Department of Statistics, Faculty of Administration and Economic, Kerbala University, Iraq

Email: zahraa.ibraheem@s.uokerbala.edu.iq

Article history:	Abstract:
Received: January 10 th 2022 Accepted: February 10 th 2022 Published: March 20 th 2022	In this paper, Binary Logistic Regression will be used to measure the probability of death due to infection with Covid-19 virus, which suffers from the presence of outliers in its measurements that represent the dependent variable of the patient's status (1 death and 0 no death) and the explanatory variables represented by the patient's age, weight, gender, diabetic ratio, the level of blood pressure, temperature and the proportion of cytokines of the two types IL-6 and IL-8, which are the cells responsible for decreasing the numbers of lymphocytes and their rise, and they have a key role in robust processes, genetic development, platelet count variable in the blood and variable blood oxygen rate. Two methods were used to estimate the Robust Binary logistic regression model, they are the Mallows Type Class method and the weighted maximum likelihood method WMLE using the Matlab language program, and the comparison between the two methods by means of the statistical criterion, mean squares of error, Akaike criterion, and Bayes Akaike criterion, to assess the best method in estimating the probability of death from infection with the virus

Keywords: Binary Logistic regression, robust, Mallows Type Class method, weighted maximum likelihood method

1. INTRODUCTION

The model in general is a mathematical representation of a problem in the real world, and the function of the model is to summarize and clarify the data as close as possible to reality, and at the same time it should be easy to understand and apply, and often the researcher has some data collected from the real world and his goal is to build a model on it without losing a lot of information. The model can consist of two different types of variables, which are the response variable (the dependent variable), which is the focus of the experiment, and it is the product of the model that the researcher wants to investigate about. The response variable can be single, or a model with two variables, or be multiple, and the explanatory variable is measured or determined. Explanatory variables by the researcher and explanatory variables are considered model inputs. These variables are called explanatory because they explain and show how the response

variable is affected by its changes. The goal of linear regression is to fit a straight line to a number of points that reduces the sum of squares of errors, that is, to fit a straight line to a number of points that reduces the sum of squares of the residuals. Regression models are used for several purposes, description and analysis of the relationship between variables, prediction and selection of variables, but when the dependent variable in the regression model is of a descriptive (qualitative) type, it takes two values in numerical form (0,1), such as (success 1, failure 0), (cure from disease 1, no cure from disease 0), (reach 1, Non-reaching 0), etc., it is called a binary Logistic Regression (BLR) model, in logistic regression the goal is not to estimate the parameters of the model (measuring the change caused by the independent variables in the dependent variable) but the goal is to measure the probability of occurrence or non-occurrence of the phenomenon under study. Sometimes, the researcher faces a situation in which



some observations in the sample under study deviate from the original format of the data, either as a result of measurement errors or as a result of mismanagement of the experiment, or sometimes the researcher is concerned that those terms go extreme or deviate from the context of the rest of the data, thus, these data can be treated as outliers (polluted), so the random variables lose one of the most important basic assumptions for them, which is the symmetry and independence of the distribution of the sample items (iid), and also if these outliers values are ignored in estimating the parameters, the variance of those estimates will increase and lead to inappropriate tests. Therefore, the application of classical methods directly to the model estimation does not give efficient estimates and thus inaccuracy in the estimation. The discovery of outliers observations are important either because those outliers themselves are important in themselves, or the experimenter wants to prevent the outliers from appearing in the estimates required. At the present time, the use of the Robust Method has grown in order to mitigate the impact of outliers on the data, which are widely used in linear models, but when the model under study is non-linear as logistic model and has a degree of complexity, these classical methods may not perform the required ones. Many researchers dealt with the logistic regression, some of them were interested in robustness and some did not pay attention to it. (Ahmad & et al, 2010) analyzes the performance of the MLE and four existing vigorous estimators under diverse exception designs. (Feng & et. al., 2014) use logistic regression with arbitrary outliers in the covariate matrix and propose a new robust logistic regression algorithm, called RoLR, that estimates the parameter through a simple linear programming procedure. (Bednarski, 2016) Computationally attractive Fisher consistent robust estimation methods based on adaptive explanatory variables trimming are proposed for the log- logistic regression model. (Dhymeaa, 2017) use of robust Bayes methods in estimating the logistic regression function on stroke disease. (Kurnaz & et al, 2018) used Robust and sparse estimation methods for linear and logistic regression in high dimensions. (Ahmed & Cheng, 2020) presents a modern course of vigorous methods for logistic regression. (S. Alshqaq & et al, 2021) examines the impact of exceptions on circular logistic regression.

2. OUTLIERS (CONTAMINATION) DATA

A contaminants or an outlier is an element that is out of the pattern characteristic of a particular set or combination (different from the normal pattern of data).

And they are data points that are far away from the majority of other data points, that is, they are observations that are not consistent with the rest of the data of the group for any of the variables for a particular phenomenon or for a group of phenomena, the value of this observation may be large or may be small located at one end of the group of observations arranged ascending or descending And that its abnormality may in many cases be a natural issue inherent in some variables. These are data that are not normally distributed. Anomalous (polluted) viewing is defined statistically as the observation that comes from a different population than the population under study. That is, the original community was polluted by observations from another community, and these observations are called contaminants. (Al-Yassery, 2007, 4). Outliers are observations that deviate greatly from other observations and are generated in a different way from the method of generating the original observations (Hekimoglu & Erenoglu, 2013, 421). While researchers (Barnett & Lewis) (1994) defined the anomalies as (observations that show up conflicting with the rest of the information set) (Obikee & et.al. 2014, 537).

3. ROBUSTNESS

The basics of the linear regression model and explained that the statistical methods are considered immune in the event of breaching the basic conditions of the methods adopted in the estimation (Dhymeaa, 2017, 17).

4. BINARY DATA:

Binary data can be defined as variables that are not subject to units of measurement and are known as qualitative variables (Cook, 2001,) , such as gender (male or female). In these cases, the binary dependent variable response (y) is either equal to one (for the occurrence of the event) or zero (the event did not occur). Since the variable (Y_i) (response variable) is Bernoulli distribution distributed, and therefore the random error term (ϵ) in the case of binary data is not distributed normally, but Bernoulli distribution is distributed with mean zero and variance $p_i(1-p_i)$ that is, it is discrete and not continuous, which causes the problem of Heterogeneity, and that the variance of the random error term depends on the logistic regression function p_i at each observation of i , which leads to the inhomogeneity of the variance of the random error term at each level of the vector x_i , and therefore it is not possible to use the method Ordinary least squares (OLS) parameter estimation process for binary data (Al-Azzawi, 2005).



5. BINARY LOGISTIC REGRESSION MODEL

Logistic regression is based mainly on the assumption that the dependent variable is a binary variable that follows the Bernoulli distribution, taking the value (1) with the probability of P (the probability of the response occurring) and the value (0) with the two probability $q=1-P$ (the probability of the response not occurring). As we know, in a linear regression that's independent and dependent variables take continuous values, the model that links the variables is:

$$Y = b_0 + b_1X_i + e \quad (1)$$

Since Y is a variable that represents a continuous variable, and the expected of the observed (real) values of Y is $E(Y/X)$ and $e=Y-\hat{Y}$ the equation (1) can be written as follows:

$$E(Y/X) = b_0 + b_1X_i \quad (2)$$

In the regression, as is known, the right-hand side of these models takes values from $(-\infty)$ to $(\infty+)$, but when the binary dependent variable takes values of zero or one, the linear regression is not appropriate because:

$$E(Y/X) = P(Y = 1) = P' \quad (3)$$

Because the value of the right-hand side is confined between zero and one. Thus, the model is not applicable from the point of view of regression, and to solve this problem, the natural logarithm is entered on the dependent variable, and since $0 \leq p \leq 1$, the ratio $P/(1-P)$ is a positive amount between $(\infty, 0)$ That is, and $0 \leq P/(1-P) \leq \infty$, therefore the regression model can be written in the case of one independent variable as follows:

$$\ln\left(\frac{P}{1-P}\right) = b_0 + b_1X_i \quad (4)$$

If we have more than one independent variable, the model is as follows:

$$\ln\left(\frac{P}{1-P}\right) = b_0 + \sum_{j=1}^k b_jX_{ij} \quad ; j = 1, 2, \dots, k \quad , \quad i = 1, 2, \dots, n \quad (5)$$

By taking the inverse of the natural logarithm of the function (5), it can be written as follows:

$$P = \frac{1}{1 + \exp(-(b_0 + \sum_{j=1}^k b_jX_{ij}))} \quad ; j = 1, 2, \dots, k \quad , \quad i = 1, 2, \dots, n \quad (6)$$

This model is called the logistic regression model or the logit model, and the transformation is called $\ln(P/(1-P))$ with the logit transformation or the logarithm Odds Ratio. And the logistic function is a continuous function

that takes the values (0,1), where y approaches zero as the right-hand side approaches $(-\infty)$ and y approaches (1) as the right-hand side approaches $(\infty+)$, and the logistic function is symmetric when The right side is equal to 1. Therefore, the logistic regression model is a logarithmic transformation of the linear regression by its transformation into a logistic function, so it will follow the characteristics of the logistic distribution, which makes the possibilities confined between (1,0), hence the name logistic regression.

6. OUTLIERS IN LOGISTIC REGRESSION:

It is imperative to recognize between the distinctive cases

of remote perceptions in logistic regression. In a binary logistic model, exemptions can happen inside the Y -space, the X -space or in both spaces. For double information, the whole y 's are 0 or 1, consequently an error within the y heading can as it were happen as a transposition $0 \rightarrow 1$ or $1 \rightarrow 0$. This sort of exception is additionally known as leftover exception or misclassification-type blunder. An perception which is extreme within the design space X is called a leverage exception or leverage point: a use point can be considered extraordinary or terrible. A good leverage point happens when $Y = 1$ with a huge value of $P(Y = \frac{1}{x_i})$ or when $Y = 0$ with little value of $P(Y = \frac{1}{x_i})$, and vice versa for a awful leverage point. Victoria-Feser (2002) showed that the MLE can be impacted by extraordinary values within the design space, and the case of misclassification errors has been studied by Pregibon (1982) and Copas (1988). Croux, et al. (2002) found that the preeminent risky special cases, named terrible use points, are misclassified observations which are at the same time distant in the design space of x variables. (Ahmad & et al, 2010, 503)

7. ROBUST LOGISTIC REGRESSION MODEL:

Let Y_i be perceptions from a Bernoulli distribution, Bernoulli(p_i); that are assumed to be created from the generalized linear model with vector of illustrative variables $x^T = (1, x_1, \dots, x_p)$, parameter vector $\beta^T = (\beta_0, \beta_1, \dots, \beta_p)$ and a function G such that $p_i = P(Y_i = 1 | X_i = x_i) = G(x_i^T \beta)$; $i = 1, 2, \dots, n$. In the logistic regression case $G(z) = ez/(1 + ez)$. The conditional probability density function (p.d.f.) of Y given x is:

$$g(y/X = x) = G(x^T \beta) \delta(y - 1) + (1 - G(x^T \beta)) \delta(y) \quad (7)$$

Where $\delta(a)$ is Dirac's measure?



Let Y_i be observations from the replacement model where it is assumed that Y_i is contaminated by another random variable $W_i \sim \text{Bernoulli}(q_i)$. Thus,

$$\hat{Y}_i = (1 - \varepsilon_{i,n})Y_i + \varepsilon_{i,n}W_i \quad (8)$$

Where the random variable $\varepsilon_{i,n} \sim \text{Bernoulli}(v/\sqrt{n})$; $v \geq 0$ and n is the sample size. The rate of contamination, $P(\varepsilon_{i,n} = 1) = v/\sqrt{n}$, is justified by contiguity of contaminated alternatives indexed by the parameter $v > 0$ with respect to the ideal model ($v = 0$).

For samples (Y_1, \dots, Y_n) and (W_1, \dots, W_n) of (iid) random variables the conditional pdf of W_i given x by the form:

$$f(y/x) = F(x)\delta(y - 1) + (1 - F(x))\delta(y) \quad (9)$$

Where $F(x) = P(W_i = 1/X_i = x_i) = q_i$, Therefore \hat{Y}_i are observations from Bernoulli distribution (\hat{p}_i) , where $\hat{p}_i = p_i + v/\sqrt{n}(q_i - p_i)$ (G.D. Mishra, & et. al., 2001, 3-4).

8. ROBUST ESTIMATORS IN LOGISTIC REGRESSION MODEL:

An exception is an perception that goes astray from the other perception values and leads to blunders within the logistic regression model. The deviation can happen in reaction factors as well as in informative factors or both. In the binary regression model, all reaction factors Y_i are parallel, taking numerical values 0 or 1, therefore, the deviation within the reaction variable can as it were happen as a $0 \rightarrow 1$ or $1 \rightarrow 0$. (Ahmed,, 2020, 130)

8.1 Mallows Type Class (Mallows)

The principle of this estimator receives the minimization of the weighted logarithm-likelihood function, where the weights depend on the illustrative variables. In this way lessening the exception value that does not compare to the other perceptions (Ahmed , Idriss Abdelmajid, Cheng , Weihu , 2020 , 10).

$$h_n(x) = [(x - \hat{\mu}_n)' \hat{\Sigma}^{-1} (x - \hat{\mu}_n)]^{\frac{1}{2}} \quad (10)$$

Where, $\hat{\mu}_n$ Robust location estimator, $\hat{\Sigma}^{-1}$ Robust variance-covariance matrix of the continuous covariates (x_1, \dots, x_n) which can be calculated by using minimum covariance determinant (MCD) approach.

Then the Mallows type estimator for logistic regression can be obtained by a solution of the form of:

$$\sum_{i=1}^n W_i [y_i \log(\prod_{i=1}^n (\beta)) + (1 - y_i) \log(1 - \prod_{i=1}^n (\beta))] \quad (11)$$

where $W_i = W(h_n(x_i))$, W is a non-increasing function such that $W(u)$ is bounded.

Bergesio, A. and Yohai, V.J. (2011) suggested choosing W depends on a constant $c > 0$.

$$W(u) = \left(1 - \frac{u^2}{c^2}\right)^3 I(|u| \leq c) \quad (12)$$

(12)

This estimate is called Mallows-type estimator estimate and the influence function of WMLE is given by: (Idriss & Cheng, 2020,132).

$$If(y, x, \beta) = M^{-1}(y - f(\beta'x))xW(h(x)) \quad (13)$$

Where, $h(x) = [(x - \mu)' \Sigma^{-1} (x - \hat{\mu}_n)]^{\frac{1}{2}}$ with μ and Σ are the limit values of $\hat{\mu}_n$ and $\hat{\Sigma}_n$,

$$M = E(W(X))F^{-1}(\beta'x)x'x \quad (14)$$

8.2 Weighted Maximum Likelihood Estimates (WMLE)

The (WMLE) method is one of the robust estimation methods proposed by (Carroll & Pederson, 1993), as it is used to estimate the parameters of the logistic regression model in the event that there are outlier's values in the data (Carroll, & Pederson, 1993).

The goal of the (WMLE) method is to find a set of values estimated to β , based on modifying the method of the maximum Likelihood method (ML) to obtaining a more efficient robust estimator by giving the observations weights to reduce the impact of outliers observations.

Assuming that we have n random variables (y_1, y_2, \dots, y_n) , then this method depends on the probability density function of random variables, which follows the Bernoulli distribution, which is according to the following formula: (Maronna & et al, 2006)

$$P(Y_i = y_i) = p_i^{y_i} (1 - p_i)^{1-y_i} \quad (15)$$

Then, the maximum likelihood function of the logistic regression model is as follows:

$$L(\beta, X) = \prod_{i=1}^n p_i^{y_i} (1 - p_i)^{1-y_i} \quad (16)$$

$$L(\beta, X) = \prod_{i=1}^n (1 - p_i) \prod_{i=1}^n \left(\frac{p_i}{1-p_i}\right)^{y_i}$$

$$L(\beta, X) = \prod_{i=1}^n (1 - p_i) \exp \left[\sum_{i=1}^n y_i \log \left(\frac{p_i}{1-p_i} \right) \right]$$

According to the logistic transfer feature, then,

$$\log \frac{p_i}{1 - p_i} = \underline{x}'_i \beta$$

$$L(\beta, X) = \prod_{i=1}^n (1 - p_i) \exp \left[\sum_{i=1}^n y_i (\underline{x}'_i \beta) \right]$$

By taking the logarithm of both sides of the over equation (16), we get the following formula:

While:

$$(1 - p_i) = \frac{1}{1 + e^{\underline{x}'_i \beta}}$$

$$\log L(\beta, X) = \left[\sum_{i=1}^n y_i (\underline{x}'_i \beta) \right] + \sum_{i=1}^n \log \left(\frac{1}{1 + e^{\underline{x}'_i \beta}} \right)$$

$$\log L(\beta, X) = \left[\sum_{i=1}^n y_i (\underline{x}'_i \beta) \right] - \sum_{i=1}^n \log (1 + e^{\underline{x}'_i \beta}) \quad (17)$$



And by equating the first derivative to the logarithm of maximum likelihood function of zero, then solving a set of equations resulting from the derivative:

$$\frac{\partial \log L(\beta, X)}{\partial \beta} = 0$$

And that the estimation in equation (17) requires following the method adopted by the (WMLE) method, which is to reduce the amount as follows: (Dhymea, 2017, 20).

$$\min \sum_{j=1}^m W_i L_i(\beta) \quad (18)$$

$$L_i(\beta) = \log L(\beta, X)$$

Where:

$L_i(\beta)$ is the logarithm of the function in the equation from equation (17)

W_i Weight function

For obtaining (WMLE) estimators, the weighted least squares method or one of the numerical methods is used as follows:

$$\hat{\beta} = (X'WX)^{-1}X'WZ \quad (18)$$

The formula (18) gives the best unbiased linear estimation of the parameter vector.as

$$Z = \begin{bmatrix} \text{Ln} \frac{p_1}{1 - p_1} \\ \text{Ln} \frac{p_2}{1 - p_2} \\ \vdots \\ \text{Ln} \frac{p_n}{1 - p_n} \end{bmatrix}$$

Where:

X the matrix of explanatory variables with a degree (m*k)

W a diagonal matrix whose main diameter elements are the weight function W_i which is represent the robust of the estimator resulting from this method depends on the weight function W_i , and this function has many forms, as the formula proposed by (Meuller and Neykov, 2003) was used using three functions of weights, namely: (Muller, and Neykov, 2003)

$$\left. \begin{aligned} W_1(t) &= (at + b), \\ W_2(t) &= (at^2 + b), \\ W_3(t) &= (a(1 - (t - 1)^6) + b), \end{aligned} \right\} \quad (19)$$

(19)

a b are constants are as follows:

$$a=0.8, b=0.2$$

t is a function that can be found according to the following formula:

$$t = h(X) = [(X - \hat{\mu})' \hat{\Sigma}^{-1} (X - \hat{\mu})]^{1/2} \quad (20)$$

$$X = [X_{i1}, X_{i2}, \dots, X_{ik}]'$$

By repeating equation (20) for m times, we get t_i as follows:

$$t_i = h(x_i) = [(x_i - \hat{\mu})' \hat{\Sigma}^{-1} (x_i - \hat{\mu})]^{1/2}; \quad i = 1, 2, \dots, n \quad (21)$$

$\hat{\mu}$ Ordinary estimates of the mean vector of k*1

$\hat{\Sigma}$ Ordinary Estimates of the Covariance Matrix of k*k

The use of this method is to improve the classical method of the (MLE) to a robust method, by following the following steps:

1. Calculate ($\hat{\mu}$) and $\hat{\Sigma}$ with the following formulas:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n x_i \quad (22)$$

$$\hat{\Sigma} = \frac{1}{n} \sum_{i=1}^n [(x_i - \hat{\mu})(x_i - \hat{\mu})'] \quad (23)$$

2. The estimates in the two equations (22) (23) are used in calculating the formula in equation (21).

3. Re-calculating both the mean and the variance matrix where ($\hat{\mu}$) and $\hat{\Sigma}$ are replaced using estimated weights. These weights depend on the values of t_i and are as follows:

$$\hat{\mu}^* = \frac{\sum_{i=1}^n w_{oi} x_i}{\sum_{i=1}^n w_{oi}} \quad (24)$$

$$\hat{\Sigma}^* = \frac{\sum_{i=1}^n w_{oi}^2 [(x_i - \hat{\mu}^*)(x_i - \hat{\mu}^*)']}{\sum_{i=1}^n w_{oi}^2} \quad (25)$$

$\hat{\mu}^*$ is robust location vector with degree k*1

$\hat{\Sigma}^*$ is robust measured matrix with degree k*k

w_{oi} Weight function

The weight function $\{w_{oi}\}$ has several formulas, and one of these formulas has been used, which is the Huber function formula), where the resulting estimator is characterized by being an efficient estimator and less sensitive to anomalies, as follows:

$$w_{oi} = \min\left\{1, \frac{g}{|t_i|}\right\} \quad (26)$$

Where $g=1.37$



w_{0i} is used in the two equations (24) (25) and then the substitution in equation (19)

The above steps represent the first iteration of the method, and then step (3) is recalculated iteratively, depending on the results of the previous iteration. The iterative process is stopped when the difference between the results of two successive iterations (when the difference between successive estimates (past and previous)) in estimating the parameters becomes little at a certain level of accuracy, it is $5e^{(-5)}$ or $5 \times 10^{(-5)}$.

REFERENCES

1. Ahmad, Sanizah; Ramli, Norazan Mohamed Midi, Habshah, (2010), "Robust Estimators in Logistic Regression: A Comparative Simulation Study", Journal of Modern Applied Statistical Methods, 1538 – 9472/10.
2. Ahmed, Idriss Abdelmajid, Cheng, Weihu, (2020), "The Performance of Robust Methods in Logistic Regression Model", Open Journal of Statistics, 10, 127-138 <https://www.scirp.org/journal/ojs> ISSN Online: 2161-7198 ISSN Print: 2161-718X, DOI: 10.4236/ojs.2020.10.
3. Al-Azzawi, Ahmed Diab (2005), "Comparison between some methods of estimating the logistic regression model and the robust methods of life experiences with binary response using the simulation method", a master's thesis in statistics, College of Administration and Economics, University of Baghdad.
4. Al-Yasiri, Tahani Mahdi Abbas, (2007), "Comparing the estimations of the robust bayes with other estimations for estimating the approximate reliability function of the Weibull distribution", PhD thesis, College of Administration and Economics - University of Baghdad.
5. Bednarski, Tadeusz, (2016), "A NOTE ON ROBUST ESTIMATION IN LOGISTIC REGRESSION MODEL". Discussions Mathematical Probability and Statistics 36 (2016) 43-51 doi:10.7151/dmps.1180.
6. Bergesio, A. and Yohai, V.J. (2011) Projection Estimators for Generalized Linear
7. Carroll, R.J. and Pederson, S. (1993), "On Robustness In The Logistic Regression Model", Journal of the Royal Statistical Society .B, Vol. 55, No. 3, pp.693-709.
8. Dhymea Hameed Shehab, 2017, "Comparison Some Robust Estimation Methods and Bayesian Method in Estimate the logistic Regression Function with Practical Application", A Thesis Submitted to The Council of College of Administration and Economics in Baghdad University.
9. Hekimoglu, Serif R.; Erenoglu, Cuneyt, (2013), "A new GM-estimate with high breakdown point", Acta Geod Geophys 48:419-437 DOI 10.1007/s40328-013-0029
10. <https://doi.org/10.1198/jasa.2011.tm09774>
11. Maronna, R.A. Martin, R. Dand Yohai, V.J. (2006). "Robust Statistics", Theory and Method, John Wiley & Sons, Ltd, The Atrium, Southern Gate, Chichester, West Sussex, England.
12. Models. Journal of the American Statistical Association, 106, 661-671.
13. Muller, Ch. H. and Neykov, N. (2003). "Breakdown Points of Trimmed Likelihood Estimators and Related Estimators in Generalized Linear Models", J. Statistic. Planning Inference 116, pp. 503-519.
14. Obikee, Adaku C., Ejuh, Godday U., Happiness; Obiora-Ilouno, (2014), "Comparison of Outlier Techniques Based on Simulated Data", Open Journal of Statistics, 4, 536-561 Published Online in SciRes.
15. S. Alshqaq, Shokrya, A. Ahmadini, Abdullah, and Abuzaid, Ali H. (2021), "Some New Robust Estimators for Circular Logistic Regression Model with Applications on Meteorological and Ecological Data", Hindawi Mathematical Problems in Engineering Volume 2021, Article ID 9944363, 15 pages <https://doi.org/10.1155/2021/9944363>.
16. Kurnaz, F. S., Hoffmann, I., & Filzmoser, P. (2018). Robust and sparse estimation methods for high-dimensional linear and logistic regression. Chemometrics and Intelligent Laboratory Systems, 172, 211-222.
17. Solomon, C., Van Rij, A. M., Barnett, R., Packer, S. G., & Lewis-Barned, N. J. (1994). Amputations in the surgical budget. The New Zealand medical journal, 107(973), 78-80.
18. Victoria-Feser, M. P. (2002). Robust inference with binary data. *Psychometrika*, 67(1), 21-32.
19. Ray, D., Yun, Y. C., Idris, M., Cheng, S., Boot, A., Iain, T. B. H., ... & Epstein, D. M. (2020). A tumor-associated splice-isoform of MAP2K7 drives dedifferentiation in MBNL1-low cancers via JNK activation. *Proceedings of the National Academy of Sciences*, 117(28), 16391-16400.
20. Stephenson, B., Cook, D., Dixon, P., Duckworth, W., Kaiser, M., Koehler, K., & Meeker, W. (2008). Binary response and logistic



World Economics & Finance Bulletin (WEFB)
Available Online at: <https://www.scholarexpress.net>
Vol. 6, January 2022,
ISSN: 2749-3628

regression analysis. available at:< a href="http://www.stat.wisc.edu/mchung/teaching/MIA/reading/GLM.logistic.Rpackage.pdf"> http://www.stat.wisc.edu/mchung/teaching/MIA/reading/GLM.logistic.Rpackage.pdf(last access: 30 August 2014).